Math 240: Determinants

Ryan Blair

University of Pennsylvania

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Outline

Today's Goals

1. Understand the definition of determinant.
2. Be able to find determinants of matrices.

Determinants of Small Matrices

The **determinant** of a square matrix is a number that determines invertibility of the matrix.

Definition

Give a 1×1 matrix $A = \begin{pmatrix} a \end{pmatrix}$, the determinant of A is

$$det(A) = \begin{vmatrix} a \end{vmatrix} = a$$

Definition

Give a 2×2 matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, the determinant of A is

$$det(A) = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

Permutations

Definition

Given the first n positive integers 1,2,...,n. A **permutation** is any arrangement of these integers in a specific order $(p_1, p_2, ..., p_n)$.

Definition

Given a permutation $(p_1, p_2, ..., p_n)$, if $p_i > p_j$ with $i < j$ we say the pair (p_i, p_j) is an **inversion**.

Definition

Let $N(p_1, p_2, ..., p_n)$ be the total number of inversions for the permutation $(p_1, p_2, ..., p_n)$. If $N(p_1, p_2, ..., p_n)$ is even we say $(p_1, p_2, ..., p_n)$ is even. If $N(p_1, p_2, ..., p_n)$ is odd we say $(p_1, p_2, ..., p_n)$ is odd.

Determinants

Definition

Let σ be a function from the set of permutations to $\{1, -1\}$ such that

$$\sigma(p_1, p_2, ..., p_n) = (-1)^{N(p_1, p_2, ..., p_n)}.$$

Definition

Let $A = (a_{i,j})$ be an $n \times n$ matrix. The **determinant** of A is

$$det(A) = \Sigma\sigma(p_1, p_2, ..., p_n)a_{1,p_1}a_{2,p_2}...a_{n,p_n}$$

where the sum is over all $n!$ distinct permutations on n numbers.

Big Theorem

Theorem

An $n \times n$ *matrix* A *is invertible if and only if* $det(A) \neq 0$.