Math 240: Linear Transformations

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Outline

Today's Goals

1. Be able to verify if a map is a linear transformation.
2. Know linear transformations of $\mathbb{R}^2$.

Mappings and Linear Transformations

Definition

Let V and W be vector spaces. A **mapping** (or **function**) from V to W is a rule that assigns to each vector v in V exactly one vector $w = T(v)$ in W. We denote this mapping by $T : V \to W$.

Definition

A mapping $T : V \to W$ is a **linear transformation** if the following hold:

1. $T(u + v) = T(u) + T(v)$ for all $u, v \in V$
2. $T(cv) = cT(v)$ for all $v \in V$ and all scalars c.

We call V the **domain** of T and W is the **codomain** of T

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Theorem

If A is an $m \times n$ matrix, then the mapping given by $T : \mathbb{R}^n \to \mathbb{R}^m$ such that $T(v) = Av$ is a linear transformation.

Matrices ARE linear transformations

Theorem

Let $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$ *be a linear transformation. T is completely described by*

$$T(v) = Av$$

where A is the $m \times n$ *matrix*

$$A = [T(\mathbf{e}_1), T(\mathbf{e}_2), ..., T(\mathbf{e}_n)]$$

and $\mathbf{e}_1, \mathbf{e}_2, ..., \mathbf{e}_n$ *are the standard basis vectors in* $\mathbb{R}^n$.

Linear Transformations from $\mathbb{R}^2$ to $\mathbb{R}^2$.

Reflections

$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Stretching

$$\begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix}$$

Shearing

$$\begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ k & 1 \end{pmatrix}$$