

Math 240: The Rank Nullity Theorem

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Outline

1 The Rank-Nullity Theorem

Today's Goals

- 1 Understand the proof of the Rank-Nullity Theorem.
- 2 Be able to apply the Rank-Nullity theorem.

The Rank-Nullity Theorem

Definition

If A is an $m \times n$ matrix, then the **null space** of A is the vector space of solutions to $Ax = 0$. In other words,

$$\text{nullspace}(A) = \{x \in \mathbb{R}^n : Ax = 0\}$$

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Theorem

(The Rank-Nullity Theorem) For any $m \times n$ matrix A ,

$$\text{rank}(A) + \text{nullity}(A) = n$$