Math 240: Row Space and Column

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Outline

Today's Goals

1. Be able to find and verify a basis of a vector space of functions.
2. Be able to find a basis for the row space and the column space of a matrix.

Spanning and Linear Independence for vector space of functions

Definition

A set of vectors $v_1, v_2, ..., v_n$ **spans** a vector space V if every vector in V can be written as $c_1 v_1 + c_2 v_2 + ... + c_n v_n$ where c_i is a scalar for $1 \leq i \leq n$.

Definition

Let $v_1, ..., v_m$ be vectors in a vector space V. The set $S = \{v_1, ..., v_m\}$ is **linearly independent** if $c_1 v_1 + c_2 v_2 + ... + c_n v_n = 0$ implies $c_1 = c_2 = ... = c_n = 0$.

Theorem

Let $f_1, ..., f_k$ be functions with continuous derivatives up to the $k-1$ order on the interval I. If the **Wronskian** *of $f_1, ..., f_k$ is non-zero at some point in I, then the set $\{f_1, ..., f_k\}$ is linearly independent on I.*

Row space and Column space

Definition

Given an $m \times n$ matrix A, the row space of A is the subspace of $\mathbb{R}^n$ spanned by the rows of A. The column space of A is the subspace of $\mathbb{R}^m$ spanned by the columns of A

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1. The row vectors of $ref(A)$ containing leading ones give a basis for the row space of A.
2. The column vectors of A corresponding to the columns of $ref(A)$ containing leading ones give a basis for the column space of A.

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2. The column vectors of A corresponding to the columns of $ref(A)$ containing leading ones give a basis for the column space of A.

Thus, $dim[rowspace(A)] = dim[columnspace(A)] = rank(A)$

Finding a basis for a subspace spanned by vectors in $\mathbb{R}^n$

We now have a new method of finding a basis for a subspace spanned by vectors in $\mathbb{R}^n$

1. Make the vectors the rows of a matrix.
2. Row reduce the matrix.
3. The row vectors of the row reduced matrix containing leading ones give a basis for the subspace spanned by the vectors.