Math 240: Basis of a Vector Space

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Outline

Today's Goals

1. Be able to find and verify a basis of a vector space.

Spanning and Linear Independence

Definition

A set of vectors $v_1, v_2, ..., v_n$ **spans** a vector space V if every vector in V can be written as $c_1 v_1 + c_2 v_2 + ... + c_n v_n$ where c_i is a scalar for $1 \leq i \leq n$.

Definition

Let $v_1, ..., v_m$ be vectors in a vector space V. The set $S = \{v_1, ..., v_m\}$ is **linearly independent** if $c_1 v_1 + c_2 v_2 + ... + c_n v_n = 0$ implies $c_1 = c_2 = ... = c_n = 0$.

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Definition

Let $\{v_1, ..., v_m\}$ be vectors in a vector space V. $\{v_1, ..., v_m\}$ is a **basis** for V if the vectors are linearly independent and span V.

Theorems Regarding Basis

Theorem

If V has a basis of m vectors, then any set of more than m vectors is linearly dependant.

Definition

The **dimension** of a vector space V is the number of vectors in any basis for V.

Theorem

If $dim(V) = n$, then any set of n vectors in V that spans V is a basis of V.

Linear Independence of Functions

Theorem

Let $f_1, ..., f_k$ be functions with continuous derivatives up to the $k-1$ order on the interval I. If the **Wronskian** *of $f_1, ..., f_k$ is non-zero at some point in I, then the set $\{f_1, ..., f_k\}$ is linearly independent on I.*