# Math 240: Spanning and Linear Independence

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# Outline

1. Spanning
2. Linear Independence

# Today's Goals

1. Be able to determine if a set of vectors spans a vector subspace.
2. Be able to determine if a set of vectors is linearly independent.

# The span of a set of vectors

**Definition**

A set of vectors $v_1, v_2, ..., v_n$ **spans** a vector space $V$ if every vector in $V$ can be written as $c_1 v_1 + c_2 v_2 + ... + c_n v_n$ where $c_i$ is a scalar for $1 \leq i \leq n$.

In this case we say $V$ is **spanned by** $v_1, v_2, ..., v_n$.

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**Exercise:** Show that $\{(1, 2, 1), (1, 0, 1), (0, 1, 1)\}$ spans $\mathbb{R}^3$.

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In this case we say $V$ is **spanned by** $v_1, v_2, ..., v_n$.

**Exercise:** Show that $\{(1,2,1),(1,0,1),(0,1,1)\}$ spans $\mathbb{R}^3$.

**Exercise:** Show that $\{(1,2,1),(1,0,1),(0,1,0)\}$ does not span $\mathbb{R}^3$.

# Linear Independence

**Definition**

Let $v_1, ..., v_m$ be vectors in a vector space $V$. The set $S = \{v_1, ..., v_m\}$ is **linearly independent** if $c_1v_1 + c_2v_2 + ... + c_nv_n = 0$ implies $c_1 = c_2 = ... = c_n = 0$.

If there exists a non trivial solution to $c_1v_1 + c_2v_2 + ... + c_nv_n = 0$ we say the set $S$ is linearly dependant.

# Linear Independence

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**Exercise:** Are the following vectors linearly independent?

$$< 1, 2, 1 >, < 1, 0, 1 >, < 0, 1, 1 >$$