Math 240: Vector Subspace

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Outline

Today's Goals

1. Understand the definition of vector subspace.
2. Be able to show a set is a vector subspace.

Vector Spaces and Vector Subspaces

Recall that a Vector space is a set of vectors together with the operations of vector addition and scalar multiplication that satisfy ten conditions.

Definition

If V is a vector space and $S \subset V$, then S is a **vector subspace** if it is a vector space under the same operations of addition and scalar multiplication as used in V.

Verifying a Vector Subspace

Theorem

If S is contained in a vector space V, then S is a subspace of V if and only if S is closed under the operations of addition and scalar multiplication in V.

Exercise: Show that the $S = \{(a, b) \in \mathbb{R}^2 : (a, b) = (2t, 3t), t \in \mathbb{R}\}$ is a subspace of $\mathbb{R}^2$.

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Exercise: Show that the $S = \{(a, b) \in \mathbb{R}^2 : (a, b) = (2t, 3t + 1), t \in \mathbb{R}\}$ is NOT a subspace of $\mathbb{R}^2$.

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Exercise: Show that if A is an $n \times n$ matrix that the set of solutions to $Ax = 0$ is a subspace of $\mathbb{R}^n$. This is called the **Null space of A**.

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Exercise: Show that the set of 2×2 matrices with determinant 1 is NOT a subspace of $M_2(\mathbb{R})$.