Math 240: Vector Space

Ryan Blair

University of Pennsylvania

Friday October 12, 2012

Outline

Today's Goals

1. Understand the definition of vector space.
2. Know examples of vector spaces.

What can you do with vectors

Vector Addition: We can add two vectors u and v together to make a new vector $u + v$.

Scalar Multiplication: We can multiply a vector u by a scalar a to make a new vector au.

What can you do with vectors

Vector Addition: We can add two vectors u and v together to make a new vector $u + v$.

Scalar Multiplication: We can multiply a vector u by a scalar a to make a new vector au.

Big Question: What makes $\mathbb{R}^n$ so special?

What can you do with vectors

Vector Addition: We can add two vectors u and v together to make a new vector $u + v$.

Scalar Multiplication: We can multiply a vector u by a scalar a to make a new vector au.

Big Question: What makes $\mathbb{R}^n$ so special?

Answer: $\mathbb{R}^n$ is a **Vector Space**!

Other Vector Spaces

1. Polynomials of degree at most n.
2. All $m \times n$ matrices.
3. All continuous functions from $\mathbb{R}$ to $\mathbb{R}$.

Definition of Vector Space

Definition

A set of vectors V is a **Real Vector Space** if V has the operations of vector addition and scalar multiplication such that the following hold for vectors u, v, w and scalars r, s:

- A1 *Closure under addition:* For every u and v in V, $u + v$ is also in V.
- A2 *Closure under scalar multiplication:* For every u in V, and scalar $k \in \mathbb{R}$, ku is also in V.
- A3 *Commutativity of addition:* For all $u, v \in V$, $u + v = v + u$.
- A4 *Associativity of Addition:* For all $u, v, w \in V$, $(u + v) + w = u + (v + w)$.
- A5 *Existence of a zero vector:* There exists a vector $\mathbf{0}$ in V such that for all $v \in V$, $\mathbf{0} + v = v$.
- A6 *Existence of additive inverses:* For each $v \in V$ there exists a vector $-v \in V$ such that $v + (-v) = \mathbf{0}$.

Definition of Vector Space (cont.)

Definition

(cont.)

- A7 *Unit Property:* For all $v \in V$, $1v{=}v$.
- A8 *Associativity of scalar multiplication:* For all $v \in V$ and all scalars $r, s \in \mathbb{R}$, $(rs)v = r(sv)$.
- A9 *Distributive property of scalar multiplication over vector addition* For all $u, v \in V$ and all scalars $r \in \mathbb{R}$, $r(u + v) = ru + rv$.
- A10 *Distributive property of scalar multiplication over scalar addition* For all $v \in V$ and all scalars $r, s \in \mathbb{R}$, $(r + s)v = rv + sv$.