Math 240: Systems of Linear Equations and Row-Echelon Form

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Outline

Systems of Linear Equations

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Linear systems are essential to finding quantitative or approximate solutions to any problem that can be stated mathematically

Vector Equations for Linear Systems

Every linear system of equations can be encoded by a vector equation.

$$Ax = b$$

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The system is **homogeneous** if b is the all zero column vector, otherwise the system is **nonhomogeneous**.

Solutions to linear systems

Not every linear system has a unique solution.

Definition

A linear system is called **consistent** if it has a solution, it is called **inconsistent** if it does not have a solution.

There are three possibilities:

1. The system has ∞-many solutions.
2. The system has a unique solution.
3. The system has no solution.

Solving a linear system

You can solve linear systems using elementary row operations.

The elementary row operations are:

1. P_{ij}: Permute row i and row j.
2. $M_i(k)$: Multiply row i by the scalar k.
3. $A_{ij}(k)$: Add k times row i to row j.

Echelon Forms

Definition

A matrix is in **row-echelon form** if

1. Any row consisting of all zeros is at the bottom of the matrix.
2. For all non-zero rows the leading entry must be a one. This is called the **leading 1**.
3. In consecutive rows the leading 1 in the lower row appears to the right of the leading 1 in the higher row.

Definition

A matrix is in **reduced row-echelon form** if it is in row-echelon form and every leading 1 is the only non-zero entry in its column.

The Row-Echelon Algorithm

Any matrix can be put in Row-Echelon form using row operations:

1. Put a leading one in the (1,1) position.
2. Use this leading one to put zeros beneath it in column 1.
3. Put a leading one in the (2,2) position.
4. So on and so forth.

The **rank** of a matrix is the number of leading ones it has when in Row-Echelon form.