MATH 240 - Spring 2011
Practice Midterm One

Name:

TA:

Recitation Time:

You may use both sides of a 8.5×11 sheet of paper for notes while you take this exam. No calculators, no course notes, no books, no help from your neighbors. Show all work, even on multiple choice or short answer questions—I will be grading as much on the basis of work shown as on the end result. Remember to put your name at the top of this page. Good luck.

Problem	Score (out of)
1	
2	
3	
4	
5	
6	
7	
Total	

1. List five properties an $n \times n$ matrix A can have that are equivalent to A being invertible.

2. Find the determinant of the following matrix.

$$
\begin{pmatrix}
-1 & 4 & 0 & 0 & 1 \\
0 & 3 & 0 & -11 & 0 \\
2 & -3 & 1 & 4 & 3 \\
0 & 0 & 0 & 5 & 0 \\
2 & 19 & 2 & 3 & -1
\end{pmatrix}
$$

3. Given the matrix A find the diagonal matrix D and the invertible matrix P such that $P^{-1}AP = D$

$$A = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & 0 \\ 1 & -1 & 1 \end{pmatrix}$$

4. For what value of a does the following system have ∞-many solutions?

$$\begin{pmatrix} 1 & -1 & 1 \\ 1 & 3 & 0 \\ 1 & 1 & a \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

5. (10 points) Write the linear map in $\mathbb{R}^3$ given by reflection about the yz-plane in terms of matrix-vector multiplication. Find the eigenvalues and eigenvectors of this map.

6. Solve the following system using matrix inverses.

$$x_1 + 2x_2 + 2x_3 = 1$$
$$x_1 - 2x_2 + 2x_3 = -3$$
$$3x_1 - x_2 + 5x_3 = 7$$

7. (Bonus!) Find the determinant of the following matrix.

$$A = (i + j)_{n \times n} \ \ for \ \ n \geq 3$$