MATH 240 - Spring 2011
Midterm Two

Name:

TA:

Recitation Time:

You may use both sides of a 8.5×11 sheet of paper for notes while you take this exam. No calculators, no course notes, no books, no help from your neighbors. Show all work, even on multiple choice or short answer questions—I will be grading as much on the basis of work shown as on the end result. Remember to put your name at the top of this page. Good luck.

Problem	**Score (out of)**
1	(10)
2	(10)
3	(10)
4	(10)
5	(5)
6	(10)
7	(10)
Total	

1. (10 pts) Find the general solution to the following differential equation.

$$y^{(6)} - y^{(5)} = 0$$

2. (10pts) Evaluate the surface integral $\int\int_S G(x,y,z)dS$ given $G(x,y,z) = (x^2+y^2)z$ and S is the portion of the sphere $x^2+y^2+z^2 = 36$ in the first octant (ie. $x \geq 0$, $y \geq 0$ and $z \geq 0$).

3. (10 pts) Solve the following IVP given $y(0) = 0$ and $y'(0) = 0$.

$$x^2y'' - 4xy' + 6y = 0$$

Concisely state why your answer does not contradict our theorem about uniqueness and existence of solutions to IVPs.

4. (10pts) Find the general solution to the following differential equation.

$$y''' + y'' + y' + 1 = e^{-x}$$

5. (5 pts) A force of 6 N stretches a spring 2 meters. A mass of 3 kg is initially released from rest from a point 2 meters below the equilibrium position, and the the subsequent motion takes place in a medium that offers a dampening numerically equal to 5 times the instantaneous velocity. Additionally, the mass is driven by an external force of $f(t) = 4cos(t)$. Write down the IVP whose solution is the equation of motion for the described situation. **Do Not Solve.**

6. (10 pts) Find all $g(y)$ such that the following line integral is path independent.

$$\int_{(0,0)}^{(12,4)} [sin(y)cos(x) + g'(y)]dx + [sin(x)cos(y) + xg'(y)]dy$$

7. Suppose C is a smooth curve bounding a region R in the plane.

A) (5 pts) Show $\frac{1}{2}\oint_C -ydx + xdy =$ Area of R. Cite any theorems you use.

B) (5 pts) Use the formula from part A) to find the area bounded by the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$. (Hint: use the parametrization $x = 2cos(t)$ and $y = 3sin(t)$).