MATH 240 - Spring 2011
Midterm One

Name:

TA:

Recitation Time:

You may use both sides of a 8.5×11 sheet of paper for notes while you take this exam. No calculators, no course notes, no books, no help from your neighbors. Show all work, even on multiple choice or short answer questions—I will be grading as much on the basis of work shown as on the end result. Remember to put your name at the top of this page. Good luck.

Problem	**Score (out of)**
1	(6)
2	(10)
3	(10)
4	(10)
5	(10)
6	(14)
7	(10)
Total	(70)

1.(6 points) Given the following matrices, find the matrix AB.

$$A = \begin{pmatrix} -2 & 3 \\ 9 & 3 \\ 5 & -5 \end{pmatrix} \quad B = \begin{pmatrix} 11 & 4 & -3 \\ 1 & -2 & 7 \end{pmatrix}$$

2.(10 points) Given the following matrices, find the determinant of the matrix $A^{-1}B$.

$$A = \begin{pmatrix} 1 & 2 & 4 \\ -3 & -3 & 1 \\ -1 & 10 & 2 \end{pmatrix} \quad B = \begin{pmatrix} 5 & 99 & -15 \\ 0 & -2 & 7 \\ 0 & 0 & 3 \end{pmatrix}$$

3.(10 points) Solve the following system using matrix inverses.

$$\begin{aligned} x_1 + 2x_2 + 2x_3 &= 2 \\ x_1 - 2x_2 + 2x_3 &= 4 \\ 3x_1 - x_2 + 5x_3 &= 8 \end{aligned}$$

4.(10 points) For what values of k is the following matrix singular?

$$\begin{pmatrix} 1 & -1 & 1 \\ 2k & 3 & 0 \\ -1 & 1 & k \end{pmatrix}$$

5. (10 points) Please mark "T" for true and "F" for false in the space provided to the left of the following statements. If the statement is false, PROVIDE A COUNTEREXAMPLE.

_____ If A is a 3×3 matrix with eigenvalues 0, 1 and 3, then A is diagonalizable.

_____ If $det(A) = 0$, then the system $Ax = 0$ has ∞-many solutions.

_____ If A and B are diagonal matrices, then $AB = BA$.

_____ If A is a square matrix and A has an eigenvalue of 0, then one of the diagonal entries of A is 0.

_____ If A and B are invertible, then $(AB)^{-1} = A^{-1}B^{-1}$.

6.(14 points) Given the matrix A find the diagonal matrix D and the invertible matrix P such that $P^{-1}AP = D$

$$A = \begin{pmatrix} -9 & 13 \\ -2 & 6 \end{pmatrix}$$

Additional space for problem 6.

7. (10 points) Suppose A is an $n \times n$ matrix such that $A^2 = A$.

What can you say about the eigenvalues of A? Explain you answer.

If, in addition, you know $det(A) = 1$, what can you say about A? Explain your answer.