Math 240: Power Series Solutions to D.E.s

Ryan Blair

University of Pennsylvania

Tuesday April 19, 2011

Outline

3 Lectures Left!

Review of Last Time

1. Used diagonalization to solve systems of linear DEs.
2. Reviewed power series basics.

Given a system $\mathbf{X}' = \mathbf{AX}$. Suppose $\mathbf{A}$ is diagnolizable(i.e. $P^{-1}AP = D$).

If $\mathbf{X} = \mathbf{PY}$ then

$$\mathbf{X} = \mathbf{P} \begin{bmatrix} c_1 e^{\lambda_1 t} \\ \vdots \\ c_n e^{\lambda_n t} \end{bmatrix}$$

Given a system $\mathbf{X}' = \mathbf{AX}$. Suppose $\mathbf{A}$ is diagnolizable(i.e. $P^{-1}AP = D$).

If $\mathbf{X} = \mathbf{PY}$ then

$$\mathbf{X} = \mathbf{P} \begin{bmatrix} c_1 e^{\lambda_1 t} \\ \vdots \\ c_n e^{\lambda_n t} \end{bmatrix}$$

Example: Solve the following system by diagonalization:

$$\mathbf{X'} = \begin{bmatrix} 1 & -1 & 2 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \mathbf{X}$$

Today's Goals

1. Find power series solutions to D.E.

Review of Power Series

Definition

$$\sum_{n=o}^{\infty} c_n(x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + \ldots$$

is a **power series centered at a**.

Definition

A function f is **analytic** at a point a if it can be represented by a power series in $x - a$ with a positive radius of convergence.

Definition

A function f is **analytic** at a point a if it can be represented by a power series in $x - a$ with a positive radius of convergence.

Definition

Given a differential equation $y'' + P(x)y' + Q(x)y = 0$, a point x_0 is an **ordinary point** if both $P(x)$ and $Q(x)$ are analytic at x_0. If a point in not ordinary it is a **singular point**.

Theorem

(Existence of power series solutions) If x_0 is an ordinary point of the differential equation $y'' + P(x)y' + Q(x)y = 0$, there are always two linearly independent power series solutions centered at x_0. In addition, each of these solutions has a radius of convergence at least the distance from x_0 to the closest singular point.

Solving D.E.s Using Power Series

Given the differential equation $y'' + P(x)y' + Q(x)y = 0$, substitute

$$y = \sum_{n}^{\infty} c_n (x-a)^n$$

and solve for the c_n to find a power series solution centered at a.