Math 240: Solving Systems of DEs by Diagonalization

Ryan Blair

University of Pennsylvania

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Outline

Review of Last Time

1. Used phase portraits to make qualitative and quantitative statements about systems.

Solutions to 2 by 2 systems

1. Two linear solutions in the phase plane implies two distinct real eigenvalues.
2. One linear solution in the phase plane implies one repeated real eigenvalue.
3. No linear solutions implies complex eigenvalues.

Today's Goals

1. Use diagonalization to solve systems of linear DEs.
2. Review power series.

Coupled and Uncoupled Systems

Definition

A system $\mathbf{X}' = \mathbf{AX}$ is **coupled** if each x_i' is expressed as a linear combination of x_1, ..., x_i.
If $\mathbf{A}$ is diagnolizable, then the system can be uncoupled, in that each x_i' can be expressed solely in terms of x_i.

Recall the following theorem from Linear Algebra

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If an $n \times n$ matrix A has n linearly independent eigenvectors, then A is diagnolizable.

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If $\mathbf{X} = \mathbf{PY}$ then

$$\mathbf{X} = \mathbf{P} \begin{bmatrix} c_1 e^{\lambda_1 t} \\ \vdots \\ c_n e^{\lambda_n t} \end{bmatrix}$$

Review of Power Series

Definition

$$\sum_{n=o}^{\infty} c_n(x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + \ldots$$

is a **power series centered at a**.

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The **radius of convergence** is the largest R such that $\sum_{n=o}^{\infty} c_n(x-a)^n$ converges for all x such that $|x-a| < R$.

Finding the Radius of Convergence

Ratio Test

Let

$$lim_{n\to\infty}\left|\frac{c_{n+1}(x-a)^{n+1}}{c_n(x-a)^n}\right| = |x-a|lim_{n\to\infty}\left|\frac{c_{n+1}}{c_n}\right| = L$$

If $L < 1$ the series converges. If $L > 1$ the series diverges. If $L = 1$ we don't know.