Math 240: The Divergence Theorem

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Outline

Review of Last Time

1. Solved flux integrals.
2. Learned about Stokes' theorem and its uses.

Stokes' Theorem

Theorem

Let S be a **piecewise smooth orientable** *surface bounded by a* **piecewise smooth simple closed** *curve C. Let $F(x,y,z) = P(x,y,z)\mathbf{i} + Q(x,y,z)\mathbf{j} + R(x,y,z)\mathbf{k}$ be a vector field* **for which P, Q and R are continuous and have continuous partial derivatives in the region of 3-space containing S.** *If C is traversed in the positive direction and $\mathbf{T}$ is the unit tangent vector to C then*

$$\oint_C F \circ d\mathbf{r} = \oint_C (F \circ \mathbf{T}) ds = \int\int_S (curl(F) \circ \mathbf{n}) dS$$

where $\mathbf{n}$ is the unit normal to S in the direction of the orientation of S.

Stokes' Theorem

Theorem

Let S be a **nice** *surface bounded by a* **nice** *curve C. Let F be a* **nice** *vector field. If C is traversed in the positive direction and* **T** *is the unit tangent vector to C then*

$$\oint_C F \circ d\mathbf{r} = \oint_C (F \circ \mathbf{T}) ds = \int\int_S (curl(F) \circ \mathbf{n}) dS$$

where **n** *is the unit normal to S in the direction of the orientation of S.*

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Let S be a **nice** *surface bounded by a* **nice** *curve C. Let F be a* **nice** *vector field. If C is traversed in the positive direction and* **T** *is the unit tangent vector to C then*

$$\oint_C F \circ d\mathbf{r} = \oint_C (F \circ \mathbf{T}) ds = \int\int_S (curl(F) \circ \mathbf{n}) dS$$

where **n** *is the unit normal to S in the direction of the orientation of S.*

Let $F =< y^2, 2z + x, 2y^2 >$. Find a plane $ax + by + cz = 0$ such that $\oint_C F \circ dr = 0$ for every closed curve C in the plane.

Today's Goals

1. Understand when and how to use the divergence theorem.

Divergence Theorem

Theorem

Let D be a **nice** *region in 3-space with* **nice** *boundary S oriented outward. Let F be a* **nice** *vector field. Then*

$$\int\int_S (F \circ \mathbf{n}) dS = \int\int\int_D div(F) dV$$

where **n** *is the unit normal vector to S.*

Divergence Theorem

Theorem

Let D be a **closed and bounded** *region in 3-space with a* **piecewise smooth** *boundary S that is oriented outward. Let $F(x, y, z) = P(x, y, z)\mathbf{i} + Q(x, y, z)\mathbf{j} + R(x, y, z)\mathbf{k}$ be a vector field* **for which P, Q and R are continuous and have continuous first partial derivatives in a region of 3-space containing D.** *Then*

$$\int\int_S (F \circ \mathbf{n})dS = \int\int\int_D div(F)dV$$

where $\mathbf{n}$ is the unit normal vector to S.