Math 240: Matrix Operations and Linear Systems

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Tuesday January 18, 2011

Outline

A Quick Review

Definition

A **matrix** is a rectangular array of numbers or functions

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

Matrix Operations

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3. Matrix multiplication: The ij entry is the dot product of the i-th row of the matrix on the left with the j-th column of the matrix on the right.

Finishing Matrix Operations

Operation: Transpose

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Notation: A^T

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Defined: Always

Performed: Rows of A become columns of A^T and columns of A become rows of A^T.

Special Matrices

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Definition

The **identity matrix of dimension** n, denoted I_n, is the $n \times n$ diagonal matrix where all the diagonal entries are 1.

Matrix Properties

Let A and B be $m \times n$ matrices. Let k and p be scalars.

1. $A + B = B + A$
2. $A + (B + C) = (A + B) + C$
3. $k(A + B) = kA + kB$
4. $(k + p)A = kA + pA$

Let 0 be the $m \times n$ matrix of all zeros

1. $A + 0 = A$
2. $A - A = 0$
3. $kA = 0$ implies $k = 0$ or $A = 0$.

More Matrix Properties

1. $A(BC) = (AB)C$
2. $A(B + C) = AB + AC$
3. $(A + B)C = AC + BC$
4. $k(AB) = (kA)B = A(kB)$
5. $I_m A = A$
6. $A I_n = A$

Even More Matrix Properties

1. $(A^T)^T = A$
2. $(kA)^T = kA^T$
3. $(A+B)^T = A^T + B^T$
4. $(AB)^T = B^T A^T$

Systems of Linear Equations

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Linear systems are essential to finding quantatative or approximate solutions to any problem that can be stated mathematically

Solutions to linear systems

Not every linear system has a unique solution.

Definition

A linear system is called **consistent** if it has a solution, it is called **inconsistent** if it does not have a solution.

There are three possibilities:

1. The system has ∞-many solutions.
2. The system has a unique solution.
3. The system has no solution.

Solving a linear system

The standard way is to use elementary operations to isolate each variable.

The elementary operations are:

1. Multiply an equation by a non-zero constant.
2. Add a non-zero multiple of one equation to another.

Echelon Forms

Definition

A matrix is in **row-echelon form** if

1. Any row consisting of all zeros is at the bottom of the matrix.
2. For all non-zero rows the leading entry must be a one. This is called the **pivot**.
3. In consecutive rows the pivot in the lower row appears to the right of the pivot in the higher row.

Definition

A matrix is in **reduced row-echelon form** if it is in row-echelon form and every pivot is the only non-zero entry in its column.

Row Operations

We will be applying row operations to augmented matrices to find solutions to linear equations. This is called **Gaussian** or **Gauss-Jordan** elimination.

Here are the row operations:

1. Multiply a row by a number.
2. Switch rows.
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Key Fact: If you alter an augmented matrix by row operations you preserve the set of solutions to the linear system.