Math 104 In Class Practice Midterm 2

Name (printed):

TA:

Recitation Time:

Please *turn off all electronic devices*. You may use both sides of a 8.5×11 sheet of paper for notes while you take this exam. No calculators, no course notes, no books, no help from your neighbors. **Show all work**, even on multiple choice or short answer questions—the grading will be based on your work shown as well as the end result. Please **clearly mark** a multiple choice option for each problem. Remember to put your name at the top of this page. Good luck.

My signature below certifies that I have complied with the University of Pennsylvania's *code of academic integrity* in completing this examination.

Your signature

Problem	**Score (out of)**
1	(10)
2	(10)
3	(10)
4	(10)
5	(10)
6	(10)
7	(10)
8	(10)
Total	(80)

1. (10 pts) Find the mean and median of the following probability density function:

$$f(x) = \begin{cases} \frac{2}{x^3} & : if \ \ x \geq 1 \\ 0 & : if x < 1 \end{cases}$$

2. (10 pts) Show that the following integral converges or show that it diverges.

$$\int_0^\infty \frac{sin^2(x)cos^2(x)}{e^x} dx$$

3. (10 pts) Evaluate the following integral or show that it does not converge.

$$\int_{-1}^{1} \frac{1}{\sqrt{|x|}} dx$$

4. (10 pts) Find the centroid of the region bounded by the x-axis and $y = cos(x)$ for $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$.

5. (10 pts) Find the volume of the object obtained by revolving the region bounded by $x = y^2$ and $x = 1 - y^2$ about the line $x = 3$.

6. (10 pts) Write the definite integral representing the volume of the object obtained by rotating the region bounded by $y = cos^4(x)$ and $y = -cos^4(x)$ about the line $x = \pi$ where the region contains the point $(0, 0)$.

7. (10 pts) Find the arc length of $y = f(x)$ from $x = -3$ to $x = 3$.

$$f(x) = \begin{cases} \sqrt{1-(x+2)^2}+1 & : if \ \ -3 \leq x \geq -1 \\ -x & : if -1 \leq x \leq 1 \\ -\sqrt{1-(x-2)^2}-1 & : if \ \ 1 \leq x \geq 3 \end{cases}$$

8. (10 pts) Let $f(x)$ be a solution to the D.E. $(y')^2 = y^2 - 1$. If $f(x) \geq 0$, show that the volume of the object obtained by rotating the region bounded by $y = f(x)$, $y = 0$, $x = a$ and $x = b$ about the x-axis is equal to half the surface area of the object obtained by rotating the curve $y = f(x)$ for $a \leq x \leq b$ about the x-axis.