Math 104: Series Convergence Tests III

Ryan Blair

University of Pennsylvania

Tuesday April 9, 2013

Ratio Test

Theorem

Given a series $\sum_{i=1}^{\infty} a_i$. *If*

$$lim_{i\to\infty}|\frac{a_{i+1}}{a_i}| = L,$$

then

1. *If* $L < 1$, *the series converges absolutely.*
2. *If* $L = 1$, *the test is inconclusive.*
3. *If* $L > 1$, *the series diverges.*

Ratio Test

Theorem

Given a series $\Sigma_{i=1}^{\infty} a_i$. *If*

$$lim_{i\to\infty}|\frac{a_{i+1}}{a_i}| = L,$$

then

1. *If* $L < 1$, *the series converges absolutely.*
2. *If* $L = 1$, *the test is inconclusive.*
3. *If* $L > 1$, *the series diverges.*

Does not help for the mundane: $\Sigma_{i=1}^{\infty} \frac{1}{i^2}$

Ratio Test

Theorem

Given a series $\sum_{i=1}^{\infty} a_i$. *If*

$$lim_{i\to\infty}|\frac{a_{i+1}}{a_i}| = L,$$

then

1. *If* $L < 1$, *the series converges absolutely.*
2. *If* $L = 1$, *the test is inconclusive.*
3. *If* $L > 1$, *the series diverges.*

Does not help for the mundane: $\sum_{i=1}^{\infty} \frac{1}{i^2}$

Helps with the crazy stuff: $\sum_{i=1}^{\infty} \frac{i^i}{i!}$

Root Test

Theorem

Given a series $\Sigma_{i=1}^{\infty} a_i$*. If*

$$lim_{i\to\infty}|a_i|^{\frac{1}{i}} = L,$$

then

1. *If* $L < 1$*, the series converges absolutely.*
2. *If* $L = 1$*, the test is inconclusive.*
3. *If* $L > 1$*, the series diverges.*

Root Test

Theorem

Given a series $\Sigma_{i=1}^{\infty} a_i$. *If*

$$lim_{i\to\infty}|a_i|^{\frac{1}{i}} = L,$$

then

1. *If* $L < 1$, *the series converges absolutely.*
2. *If* $L = 1$, *the test is inconclusive.*
3. *If* $L > 1$, *the series diverges.*

Helps with series involving variable powers: $\Sigma_{i=1}^{\infty}\left(\frac{i}{3i+4}\right)^i$

Root Test

Theorem

Given a series $\Sigma_{i=1}^{\infty} a_i$. *If*

$$lim_{i\to\infty}|a_i|^{\frac{1}{i}} = L,$$

then

1. *If* $L < 1$, *the series converges absolutely.*
2. *If* $L = 1$, *the test is inconclusive.*
3. *If* $L > 1$, *the series diverges.*

Helps with series involving variable powers: $\Sigma_{i=1}^{\infty}(\frac{i}{3i+4})^i$

Helps with series involving variable powers: $\Sigma_{i=1}^{\infty} i(\frac{2}{3})^i$

Rules for testing convergence of $\sum_{i=1}^{\infty} a_i$ when a_i is positive

1. Try the nth term test first.
2. If you can integrate a_i, use the Integral Test.
3. If a_i is a rational function in i, use Limit Comparison.
4. If you can see that a known series is "close to" your series, but you can't see how to relate them using inequalities, use Limit Comparison test.
5. If the terms of a series contain factorials, try to use the Ratio Test.
6. If the general term a_i contains products or quotients of terms with i appearing in the exponent, try to use the Root Test.

Examples

$$\sum_{n=1}^{\infty}\left(\frac{1}{n^2}+1.01^n\right)$$
$$\sum_{n=1}^{\infty}\left(\arcsin\frac{1}{n}\right)^n$$
$$\sum_{n=2}^{\infty}\frac{1}{n(\ln n)^2}$$
$$\sum_{n=1}^{\infty}\frac{3^n}{n\cdot 2^n}$$

More Examples

$$\sum_{n=1}^{\infty} \frac{4n^3 + 5}{7n^2 - 11n^3}$$

$$\sum_{n=1}^{\infty} \frac{n^n}{3^n \cdot n!}$$

$$\sum_{n=1}^{\infty} \frac{3^n + n^3}{7^n + 10}$$

$$\sum_{n=1}^{\infty} \frac{1 + e^{-n^3}}{n}$$