Math 104: Series Convergence Tests II

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Limit Comparison Test

Theorem

Let $\Sigma_{i=1}^{\infty} a_i$ *and* $\Sigma_{i=1}^{\infty} b_i$ *be positive series. If*

$$lim_{i\to\infty}\frac{a_i}{b_i} = C$$

where C *is a finite positive constant, then either both* $\Sigma_{i=1}^{\infty} a_i$ *and* $\Sigma_{i=1}^{\infty} b_i$ *converge or both* $\Sigma_{i=1}^{\infty} a_i$ *and* $\Sigma_{i=1}^{\infty} b_i$ *diverge.*

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Determine if $\Sigma_{i=1}^{\infty}\frac{i+2}{(i+1)^3}$ is convergent or divergent.

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Determine if $\Sigma_{i=1}^{\infty}\frac{i+2}{(i+1)^3}$ is convergent or divergent.

Determine if $\Sigma_{i=1}^{\infty}\frac{2i^2-1}{i^2 3^i}$ is convergent or divergent.

Alternating Sequences

An **alternating** series is of the form $\Sigma_{i=1}^{\infty} a_i$ where $a_i = (-1)^i b_i$ or $a_i = (-1)^{i+1} b_i$ where $b_i \geq 0$ for all i.

Theorem

(Alternating Series Test)
If the alternating series $\Sigma_{i=1}^{\infty} a_i$ satisfies

1. $b_{i+1} \leq b_i$ *for all* i
2. $lim_{i \to \infty} b_i = 0.$

Then $\Sigma_{i=1}^{\infty} a_i$ converges.

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Determine the convergence or divergence of $\Sigma_{i=1}^{\infty} (-1)^i \frac{n}{n+1}$.

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Determine the convergence or divergence of $\Sigma_{i=1}^{\infty} \frac{(-1)^i}{i}$.
Determine the convergence or divergence of $\Sigma_{i=1}^{\infty} (-1)^i \frac{n}{n+1}$.
Determine the convergence or divergence of $\Sigma_{i=1}^{\infty} cos(n\pi) \frac{1}{n^{\frac{2}{3}}}$.

Conditional and Absolute Convergence

Definition

A series $\Sigma_{i=1}^{\infty} a_i$ is **absolutely** convergent if $\Sigma_{i=1}^{\infty} |a_i|$ is convergent.

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A series $\Sigma_{i=1}^{\infty} a_i$ is **conditionally** convergent if $\Sigma_{i=1}^{\infty} |a_i|$ is divergent and $\Sigma_{i=1}^{\infty} a_i$ is convergent.

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Determine conditional or absolute convergence of $\Sigma_{i=1}^{\infty} \frac{(-1)^i}{i}$.

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Definition

A series $\sum_{i=1}^{\infty} a_i$ is **absolutely** convergent if $\sum_{i=1}^{\infty} |a_i|$ is convergent.

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A series $\sum_{i=1}^{\infty} a_i$ is **conditionally** convergent if $\sum_{i=1}^{\infty} |a_i|$ is divergent and $\sum_{i=1}^{\infty} a_i$ is convergent.

Determine conditional or absolute convergence of $\sum_{i=1}^{\infty} \frac{(-1)^i}{i}$.

Determine conditional or absolute convergence of $\sum_{i=1}^{\infty} \frac{cos(i)}{i^2}$.

Ratio Test

Theorem

Given a series $\Sigma_{i=1}^{\infty} a_i$. *If*

$$lim_{i \to \infty} \left| \frac{a_{i+1}}{a_i} \right| = L,$$

then

1. *If* $L < 1$, *the series converges absolutely.*
2. *If* $L = 1$, *the test is inconclusive.*
3. *If* $L > 1$, *the series diverges.*

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Helps with the crazy stuff: $\sum_{i=1}^{\infty} \frac{i^i}{i!}$

Root Test

Theorem

Given a series $\Sigma_{i=1}^{\infty} a_i$. *If*

$$lim_{i\to\infty}|a_i|^{\frac{1}{i}} = L,$$

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