Math 104: Practice Midterm 3

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Final Exam

1. Wednesday May 1st from noon until 2pm.
2. Bring Penn ID and an 8.5x11 hand written page of notes.
3. 15 multiple choice problems (8 on integration and applications, 5 on series and sequences, 2 on differential equations).

Studying for the final

1. My Office hours Thursday 3pm to 4pm, Tuesday 5pm to 7pm
2. Other professors review sessions are open to you.
3. Use the finals from spring 2011, fall 2011, spring 2012 and fall 2012 as your practice finals

Problem 1

Show that the following sequence converges to zero.

$$\{\frac{cos^2(n)}{2^n}\}$$

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Theorem

(Squeeze) Given sequences a_n, b_n and c_n such that $a_n \leq b_n \leq c_n$ for all n and $lim_{n\to\infty} a_n = lim_{n\to\infty} c_n = L$, then

$$lim_{n\to\infty} b_n = L$$

Problem 2

Determine if the following series converges. If it does, find the value of the sum.

$$\sum_{n=1}^{\infty} \frac{3}{n(n+3)}$$

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$$\Sigma_{n=1}^{\infty} \frac{3}{n(n+3)}$$

Definition

The **n-th partial sum** for a sequence $\{a_i\}_{i=1}^{\infty}$ is

$$S_n = \Sigma_{i=1}^{n} a_i = a_1 + a_2 + a_3 + ... + a_n$$

Definition

A **Series**

$$\Sigma_{i=1}^{\infty} a_i = lim_{n \to \infty} S_n$$

Problem 3

For what values of p is the following series convergent?

$$\Sigma_{n=2}^{\infty} \frac{1}{n(ln(n))^p}$$

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Theorem

Let f be a continuous, positive, decreasing function on $[c, \infty]$. If $a_i = f(i)$, then

1. *If $\int_c^\infty f(x)dx$ is convergent, then $\Sigma_{i=c}^{\infty} a_i$ is convergent.*
2. *If $\int_c^\infty f(x)dx$ is divergent, then $\Sigma_{i=c}^{\infty} a_i$ is divergent.*

Problem 4

Determine if the following series converges or diverges.

$$\Sigma_{n=1}^{\infty} sin(\frac{1}{n})$$

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Theorem

Let $\Sigma_{i=1}^{\infty} a_i$ and $\Sigma_{i=1}^{\infty} b_i$ be positive series. If

$$lim_{i\to\infty}\frac{a_i}{b_i} = C$$

where C is a finite positive constant, then either both $\Sigma_{i=1}^{\infty} a_i$ and $\Sigma_{i=1}^{\infty} b_i$ converge or both $\Sigma_{i=1}^{\infty} a_i$ and $\Sigma_{i=1}^{\infty} b_i$ diverge.

Problem 5

Show that the following series is conditionally convergent.

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{ln(n)}$$

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Theorem

(Alternating Series Test)
If the alternating series $\Sigma_{i=1}^{\infty} a_i$ *satisfies*

1. $b_{i+1} \leq b_i$ *for all* i
2. $lim_{i \to \infty} b_i = 0.$

Then $\Sigma_{i=1}^{\infty} a_i$ *converges.*

Problem 6

For what values of k is the following series divergent?

$$\sum_{n=1}^{\infty} \frac{(n!)^2}{(kn)!}$$

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$$\Sigma_{n=1}^{\infty} \frac{(n!)^2}{(kn)!}$$

Theorem

Given a series $\Sigma_{i=1}^{\infty} a_i$. *If*

$$lim_{i\to\infty} |\frac{a_{i+1}}{a_i}| = L,$$

then

1. *If* $L < 1$*, the series converges absolutely.*
2. *If* $L = 1$*, the test is inconclusive.*
3. *If* $L > 1$*, the series diverges.*

Problem 7

Find the interval of convergence for the following power series.

$$\Sigma_{n=1}^{\infty} \frac{n(2x)^n}{(n^2+1)3^n}$$

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$$R = lim_{k\to\infty} \left| \frac{c_k}{c_{k+1}} \right|$$

Given a power series $\Sigma_{k=0}^{\infty} c_k (x-a)^k$ with radius of convergence R, the **interval of convergence** is one of the following where we include endpoints it the series convergence at those points.

$$(a-R, a+R), [a-R, a+R), (a-R, a+R], [a-R, a+R]$$

Problem 8

Bound the error in approximating e^x on the interval $[-4, -2]$ using the first six terms in its Maclaurin series.

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Theorem (Taylor's Theorem)

Given a Taylor Series $\sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k$*, if there is a constant* M *such that* $|f^{(n+1)}(t)| < M$ *for all* t *between* a *and* x*, then* $|R_n(x)| < M\frac{|x-a|^{n+1}}{(n+1)!}$