Math 104: Series Convergence Tests

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Tuesday April 2, 2013

Series in terms of Sequences

Roughly, an infinite series $\Sigma_{i=1}^{\infty} a_i$ denotes the sum of the terms in the sequence $\{a_i\}_{i=1}^{\infty}$.

Definition

The **n-th partial sum** for a sequence $\{a_i\}_{i=1}^{\infty}$ is

$$S_n = \Sigma_{i=1}^{n} a_i = a_1 + a_2 + a_3 + ... + a_n$$

Definition

A **Series**

$$\Sigma_{i=1}^{\infty} a_i = lim_{n \to \infty} S_n$$

Nth Term Test for Convergence

Theorem

If a series $\sum_{i=1}^{\infty} a_i$ *converges then* $lim_{i \to \infty} a_i = 0$.

Theorem

If $lim_{i \to \infty} a_i \neq 0$ *or does not exist, then* $\sum_{i=1}^{\infty} a_i$ *diverges.*

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If a series $\Sigma_{i=1}^{\infty} a_i$ converges then $lim_{i \to \infty} a_i = 0$.

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If $lim_{i \to \infty} a_i \neq 0$ or does not exist, then $\Sigma_{i=1}^{\infty} a_i$ diverges.

Exercise: Determine the convergence or divergence of $\Sigma_{i=1}^{\infty} ln(\frac{i^2+1}{2i^2+1})$

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Exercise: Determine the convergence or divergence of $\Sigma_{i=1}^{\infty} \frac{e^i}{i^2}$.

Integral Test

Theorem

Let f be a continuous, positive, decreasing function on $[c, \infty]$. If $a_i = f(i)$, then

1. *If $\int_c^\infty f(x)dx$ is convergent, then $\Sigma_{i=c}^\infty a_i$ is convergent.*
2. *If $\int_c^\infty f(x)dx$ is divergent, then $\Sigma_{i=c}^\infty a_i$ is divergent.*

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Show that $\Sigma_{i=1}^\infty \frac{1}{i^p}$ is convergent for $p > 1$.

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Show that $\Sigma_{i=1}^{\infty} \frac{1}{i^p}$ is convergent for $p > 1$.
Determine if $\Sigma_{i=2}^{\infty} \frac{1}{iln(i)}$ is convergent or divergent.

The Direct Comparison Test

Theorem

Let $\sum_{i=1}^{\infty} a_i$ and $\sum_{i=1}^{\infty} b_i$ be positive series.

1. *If $a_i \leq b_i$ for all i and $\sum_{i=1}^{\infty} b_i$ converges, then $\sum_{i=1}^{\infty} a_i$ converges.*
2. *If $a_i \leq b_i$ for all i and $\sum_{i=1}^{\infty} a_i$ diverges, then $\sum_{i=1}^{\infty} b_i$ diverges.*

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Determine if $\sum_{i=1}^{\infty} \frac{1}{i!}$ is convergent or divergent.

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Determine if $\Sigma_{i=1}^{\infty} \frac{1}{i!}$ is convergent or divergent.

Determine if $\Sigma_{i=2}^{\infty} \frac{\sqrt{i}}{2i-1}$ is convergent or divergent.

Limit Comparison Test

Theorem

Let $\Sigma_{i=1}^{\infty} a_i$ and $\Sigma_{i=1}^{\infty} b_i$ be positive series. If

$$lim_{i\to\infty}\frac{a_i}{b_i} = C$$

where C is a finite positive constant, then either both $\Sigma_{i=1}^{\infty} a_i$ and $\Sigma_{i=1}^{\infty} b_i$ converge or both $\Sigma_{i=1}^{\infty} a_i$ and $\Sigma_{i=1}^{\infty} b_i$ diverge.

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Determine if $\Sigma_{i=1}^{\infty} \frac{i+2}{(i+1)^3}$ is convergent or divergent.

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Determine if $\Sigma_{i=1}^{\infty}\frac{i+2}{(i+1)^3}$ is convergent or divergent.

Determine if $\Sigma_{i=1}^{\infty}\frac{2i^2-1}{i^2 3^i}$ is convergent or divergent.