Math 104: Calculating Surface Area

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Outline

Shells Method

The general formula for calculating volumes of revolution using the shell method.

Vol $= \int_a^b$ volume of the shell slice

When rotating about the y-axis we get

Vol $= \int_a^b 2\pi($ radius of shell$)($height of shell$)dx$

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Find the volume of the solid obtained by rotating the region in the xy-plane bounded by
$y = x^3$ $y = 0$ $x = 2$
about the y-axis.

Areas of surfaces of revolution

Definition

The **Area of the surface** generated by revolving the graph of $y = f(x)$ about the x-axis is

$$SA = \int_a^b 2\pi f(x)\sqrt{1 + (\frac{dy}{dx})^2}dx$$

The **Area of the surface** generated by revolving the graph of $x = g(y)$ about the y-axis is

$$SA = \int_a^b 2\pi g(y)\sqrt{1 + (\frac{dx}{dy})^2}dx$$

Examples

Example 1: Find the area of of the object obtained from rotating the curve $y = \sqrt{4 - x^2}$ from $x = -1$ to $x = 1$ about the x-axis.

Example 2: Find the area of of the object obtained from rotating the curve $y = x^{\frac{1}{3}}$ from $x = 0$ to $x = 1$ about the y-axis.

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Example 3: Find the area of the object obtained from rotating the curve $y = \frac{1}{x}$ from $x = 1$ to $x = b$ about the x-axis.

Example 4: Find the volume of the solid obtained from rotating the region bounded by $y = \frac{1}{x}$, $x = 1$, $x = b$ and $y = 0$ about the x-axis.

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Example 3: Find the area of the object obtained from rotating the curve $y = \frac{1}{x}$ from $x = 1$ to $x = b$ about the x-axis.

Example 4: Find the volume of the solid obtained from rotating the region bounded by $y = \frac{1}{x}$, $x = 1$, $x = b$ and $y = 0$ about the x-axis.

Find the limit of these values as b goes to infinity.