Math 104: Volumes Cont.

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Outline

1 Volumes of Rotation

Volumes of solids of rotation

Replace all x's with y's in the following formulas to get other valid expressions for volume.

Disks:

$\text{Vol} = \int_a^b \pi(\text{radius in terms of x})^2 dx$

Shells:

$\text{Vol} = \int_a^b 2\pi(\text{radius in terms of x})(\text{height in terms of x}) dx$

Washers:

$\text{Vol} =$

$\int_a^b \pi(\text{outer radius in terms of x})^2 - \pi(\text{inner radius in terms of x})^2 dx$

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Exercise: Find the volume of the object obtained by rotating the region bounded by the lines $y = x$, $y = 1$ and $x = 0$ about the x-axis.

Calculating Volumes of rotation

Let R_1 be the region in the xy-plane bounded by

$$y = x^3$$

$$x = 0$$

$$y = 8$$

Let R_2 be the region in the xy-plane bounded by

$$y = x^3$$

$$y = 0$$

$$x = 2$$

Calculating Volumes of rotation

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Let R_2 be the region in the xy-plane bounded by

$$y = x^3$$

$$y = 0$$

$$x = 2$$

Exercise: Find the volume of the region R_2 rotated about the x-axis.

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$$x = 0$$

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Let R_2 be the region in the xy-plane bounded by

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Exercise: Find the volume of the region R_1 rotated about the line $y = 8$.

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Exercise: Find the volume of the region R_2 rotated about the $y = 8$.

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Exercise: Find the volume of the region R_1 rotated about the y-axis.

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Exercise: Find the volume of the region R_2 rotated about the y-axis.

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$$x = 2$$

Exercise: Find the volume of the region R_1 rotated about the line $y = -1$.

Infinite Shape

Find the volume of the solid obtained by rotating the region in the xy-plane bounded by

$$x = 1$$

$$y = 0$$

$$x = R$$

$$y = \frac{ln(x)}{\sqrt{x}}$$

about the x-axis.

Shells Method

Find the volume of the solid obtained by rotating the region in the xy-plane bounded by

$$y = 3x - x^2$$

$$y = 0$$

about the y-axis.

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Find the volume of the solid obtained by rotating the region in the xy-plane bounded by

$$y = 3x - x^2$$

$$y = 0$$

about the y-axis.
Shells is much easier than washers for this problem