Math 104: Volumes

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Outline

Area between Curves

Find the area between the following two curves in two ways

$$y = \frac{1}{2}x$$

$$y^2 = 8 - x$$

First, by integrating with respect to y.
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1. Sketch the graphs and label points of intersection.
2. Write down formulas for the upper and lower boundaries of the region.
3. Integrate the difference of the formulas over the appropriate intervals.

Volume Basics

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Example: Find the volume of the box with dimensions

$$0 \leq x \leq 2$$

$$1 \leq y \leq 3$$

$$2 \leq z \leq 4$$

in three different ways by integrating with respect to x, then y, then z.

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Exercise 3: Slicing into vertical ... shells.

Volume of a hyperboloid

Find the volume of the solid obtained by rotating the curve $y = x^2$ from $x = -2$ to $x = 2$ about the y-axis by

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Exercise 2: Slicing into vertical shells.

Volumes of solids of rotation

Replace all x's with y's in the following formulas to get other valid expressions for volume.

Disks:

$\text{Vol} = \int_a^b \pi(\text{radius in terms of x})^2 dx$

Shells:

$\text{Vol} = \int_a^b 2\pi(\text{radius in terms of x})(\text{height in terms of x}) dx$

Washers:

$\text{Vol} =$

$\int_a^b \pi(\text{outer radius in terms of x})^2 - \pi(\text{inner radius in terms of x})^2 dx$

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Exercise: Find the volume of the object obtained by rotating the region bounded by the lines $y = x$, $y = 1$ and $x = 0$ about the x-axis.