Math 104: Applications of Definite Integrals

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Outline

Types of integrals

Indefinite Integrals represent families of antiderivatives

$$\int x dx = \frac{x^2}{2} + c$$

Indefinite integrals are useful for solving differential equations.

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Definite Integrals represent the area under the curve

$$\int_0^2 x dx = 2$$

Definite integrals are useful for solving problems is Geometry, Physics and Statistics.

Definition of Definite Integral

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$$\int_a^b f(x)dx = lim_{n\to\infty}\Sigma_{i=1}^n f(a + \frac{b-a}{n}i)\frac{b-a}{n}$$

Fundamental theorem of calculus

Theorem

Let $f(x)$ be a continuous function with antiderivative $G(x)$

1. $$\frac{d}{dx}\left(\int_a^x f(t)dt\right) = f(x)$$

2. $$\int_a^b f(x)dx = G(b) - G(a)$$

The big idea:

$$\int d\ddot{\smile} = \ddot{\smile}$$

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Example: Find circumference of the circle $x^2 + y^2 = 4$.

Calculating area in different coordinates

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Exercise: Calculate the area of the disk in three different ways: using wedges, using circular bands and using vertical bands