Math 104: Euler's Method and Applications of ODEs

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Outline

1. Review
2. Euler's Method
3. Mixing Problems

Review

Last time we learned how to solve certain differential equations using

1. separation of variables.
2. integrating factor method.

We also learned how to visualized first order ODEs using slope fields.

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Exercise: Solve the following differential equation $y' + xy = x$.

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Exercise: Solve the following differential equation $y' + xy = x$.
Exercise: Graph the slope field of $y' + xy = x$ and use it to find the behavior at infinity of the solution to the IVP $y' + xy = x$ and $y(0) = -2$.

Euler's Method

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Warm-up Example: Let $f(x)$ be a solution to the IVP $\frac{dy}{dx} = x + y, y(0) = -1$. Find a linear approximation of $f(x)$ at $x = 0$ and use this to estimate $f(1)$

Euler's Method

Approximating solutions to $\frac{dy}{dx} = F(x, y), y(x_0) = y_0$.

1. Choose a step size h.
2. Derive a linear approximation $y_1 = y_0 + hF(x_0, y_0)$.
3. Take one "step" and derive the second approximation at $x_1 = x_0 + h$ given by $y_2 = y_1 + hF(x_1, y_1)$.
4. Continue inductively to derive additional approximations $x_{k+1} = x_k + h$ and $y_{k+1} = y_k + hF(x_k, y_k)$

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Example: Let $f(x)$ be a solution to the IVP $\frac{dy}{dx} = y, y(0) = 1$. Use Euler's Method to approximate $f(1)$ with a step size of $\frac{1}{3}$.

Mixing Problems

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Example: A tank contains 15kg of salt dissolved in 1000L of water. Brine that contains .05kg of salt per liter is pumped in at a rate of 10L per minute.

The solution is thoroughly mixed and drained at a rate of 10L per minute.

What is the amount of salt in the tank at t minutes?

Mixing Problems

A vat with 500L of beer contains 4 percent alcohol.
Beer with 6 percent alcohol is poured in at a rate of 5L per min.
The mixture is pumped out at the same rate.
What is the percent of alcohol after one hour?