Math 104: l'Hospital's rule, Differential Equations and Integration

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Tuesday January 22, 2013

Outline

l'Hospital's rule

Last time we saw how to find certain $\frac{0}{0}$ limits using Taylor Series.

Theorem

If

$$lim_{x\to a} f(x) = 0 = lim_{x\to a} g(x),$$

Then

$$lim_{x\to a} \frac{f(x)}{g(x)} = lim_{x\to a} \frac{f'(x)}{g'(x)}.$$

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Exercise: Find the following limit using l'Hospital's

$$lim_{x\to 0}\frac{e^x - 1 - x}{x^2}$$

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Exercise: Use Taylor series to prove l'Hospital's theorem supposing $\frac{f'(a)}{g'(a)}$ is well defined

Differential Equations

Definition

An nth order Ordinary Differential Equation (O.D.E.) on $y(x)$ is an algebraic equation including y and its derivatives

$$F(x, y(x), \frac{dy}{dx}, \frac{d^2y}{dx^2}, ..., \frac{d^ny}{dx^n}) = 0$$

A solution is any **function** that satisfies the above equation.

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Example: An object near the surface of the earth acted on by gravity has velocity $v(t)$ given by $\frac{dv}{dt} = v'(t) = -g$
Solve for $v(t)$.

Initial value problem

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Example: Solve the following IVP
$\frac{dy}{dx} = cos(2x)$ and $y(0) = -1$.

Summary of Integration Techniques

To solve O.D.E.s we need to be able to integrate.

1. u substitution
2. integration by parts
3. trigonometric substitutions
4. partial fractions
5. completing the square

U-Substitution

$$\int f(g(x))g'(x)dx = \int f(u)du$$

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Exercise Find $\int x^2 e^{x^3} dx$.

Exercise Find $\int \frac{(1+ln(x))^3}{x} dx$.

Integration by Parts

$$\int u(x)v'(x)dx = u(x)v(x) - \int u'(x)v(x)dx$$

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Example: Find $\int xe^x dx$.