Math 104: Taylor series, Limits and l'Hospital's rule

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University of Pennsylvania

Tuesday January 15, 2013

Outline

Welcome

Adding the Course

Speak to Robin Toney in the Math office on the 4th floor of DRL.

Space is limited.

Syllabus Highlights

Course Webpage:
http://www.math.upenn.edu/∼ryblair/Math104/index.html

Here you will find

1. Lecture slides
2. Homework assignments
3. A copy of the syllabus
4. A link to Blackboard (were your quiz homework and test scores are posted)
5. Other useful links

Email

1. Include Math 104 in the subject line
2. Send it from a Penn account
3. The body should include your name and your recitation number
4. Allow 24 hrs for a reply
5. Direct quiz questions to your TA, everything else to me

Text

Required Text: Thomas' Calculus, Custom Edition for the University of Pennsylvania.

ISBN 13: 978-1-256-32659-5.

Grading

1. 20% Homework (10% online and 10% Handed in)
2. 20% Quizzes
3. 15% Midterm 1
4. 15% Midterm 2
5. 30% Final

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Course grades are curved using the final exam in accordance with the math departments 30-30-30-10 policy.

Written Homework

1. Written homework will be assigned each week based on the material covered that week.
2. You can find the current homework assignment on the course website.
3. Homework will be collected in recitation.
4. The first written Homework will be posted tonight and due on Jan 21 or Jan 23.

Online Homework

1. Online Homework will be assigned each week based on the material covered that week.
2. You complete online homework through math lab here is a link:

http://portal.mypearson.com/mypearson-login.jsp

Quiz

1. There will be a quiz in each recitation.
2. Quiz questions will be based on the homework assigned the previous week.

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2. Quiz questions will be based on the homework assigned the previous week.
3. **Next week's quiz question will be based on the material found in the syllabus.**

Exams

Mark your calendars

1. Midterm 1: Feb. 12
2. Midterm 2: CHANGED TO Mar. 21
3. Final: May 1

Classroom Decorum:(Common Courtesy)

1. No Talking
2. No Texting
3. Cellphone Ringers Off
4. Laptops only used for taking notes

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If these constraints are too much, feel free to step outside.

Taylor Series

Definition

The **Taylor series** generated by a function f at $x = a$ is

$$\Sigma_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + \ldots$$

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Exercise: Verify that the Taylor series of e^x at $x = 0$ is $\Sigma_{k=0}^{\infty} \frac{x^k}{k!}$

Taylor Series are closely related to approximations

Example: Graph the following functions side-by-side:

- e^x
- 1
- $1 + x$
- $1 + x + \frac{x^2}{2}$
- $1 + x + \frac{x^2}{2} + \frac{x^3}{6}$

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Core Idea: A Taylor Series is the LIMIT of successively better polynomial approximations!

Tricks to finding Taylor Series

Problem: Find the Taylor series for $f(x) = ln(x+1)$ at $x = 0$.

Trick: No trick, just substitute into the formula for Taylor series and find the pattern.

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Answer: $\Sigma_{k=1}^{\infty}(-1)^{k-1}\frac{x^k}{k}$

Tricks to finding Taylor Series

Problem: Find the Taylor series for $f(x) = ln(x)$ at $x = 1$.

Trick: Save yourself time and use the Taylor Series we just found.

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Trick: Save yourself time and use the Taylor Series we just found.

Answer: $\Sigma_{k=1}^{\infty}(-1)^{k-1}\frac{(x-1)^k}{k}$

Tricks to finding Taylor Series

Problem: Find the first 3 terms of the Taylor series for $f(x) = xsin(3x)$ at $x = 0$.

Trick: Use the fact that you know that Taylor Series for sin(x).

Tricks to finding Taylor Series

Problem: Find the first 3 terms of the Taylor series for $f(x) = e^x sin(x)$ at $x = 0$.

Trick: Use the fact that you know that Taylor Series for $sin(x)$ and you know the Taylor Series for e^x.