Math 103: Big Concepts Review

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Thursday December 8, 2011

Outline

Final Exam Announcements

1. On Wednesday December 14th from 9 am to 11 am.
2. The exam is accumulative.
3. Last names beginning with A, B, C or D report to Cohen Hall 402.
4. Everyone else takes the exam in Cohen Hall 17.
5. Allowed one 8.5 by 11 sheet of notes front and back
6. Bring your Penn ID.
7. The final is 15 questions.

What to study for the final

1. Old final exams (http://www.math.upenn.edu/ugrad/calc/m103/oldexams.html)
2. Our practice midterms and midterms (http://www.math.upenn.edu/ ryblair/Math103F11/index.html)
3. Homework problems and examples done in class.
4. Extra office hours Monday from 4 to 6 pm and Tuesday from 4 to 6pm.

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Notation. Other ways of writing the derivative of $y = f(x)$.

$$f'(x) = y' = \frac{dy}{dx} = \frac{df}{dx} = \frac{d}{dx} f(x) = Df(x) = D_x f(x)$$

The Intermediate Value Theorem and Mean Value Theorem

Theorem

Suppose $f(x)$ is continuous on $[a, b]$ and if y_0 is any value between $f(a)$ and $f(b)$, then $y_0 = f(c)$ from some c in $[a, b]$.

Theorem

Suppose $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b). Then there exists a number c such that $a < c < b$ and

$$\frac{f(b) - f(a)}{b - a} = f'(c).$$

L'Hospital's Rule for $\frac{0}{0}$

Theorem

Suppose $f(a) = g(a) = 0$, f and g are differentiable near a and $g'(x) \neq 0$ for x near a but not equal to a, Then

$$lim_{x \to a} \frac{f(x)}{g(x)} = lim_{x \to a} \frac{f'(x)}{g'(x)}$$

if the right-hand limit exists.

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6. Identify any asymptotes.
7. Plot key points and asymptotes, and sketch the curve.

Example

A boat is being pulled toward a dock by a rope from the bow of the boat to the bottom of the dock which is 6 ft above the the bow. If the rope is hauled at a rate of 2 $\frac{ft}{sec}$, at what rate is the angle θ changing when there is 10 ft of rope between the boat and the dock.

1. Draw a picture representing the problem.
2. Introduce variables and find a formula for the quantity being optimized.
3. Use the information in the problem to express the quantity being optimized in terms of a single variable.
4. Use the first derivative test to find the local minima and maxima.
5. Finish solving the problem.

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How To Approach These Problems

1. Draw a picture and name the variables and constants.
2. Write down any additional numerical info.
3. Write down what you are asked to find.
4. Write an equation that relates the quantities.
5. Differentiate with respect to t.
6. Finish solving the problem. Remember units.

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Theorem

If f is integrable on $[a, b]$, then

$$\int_a^b f(x)dx = lim_{n \to \infty} \sum_{i=1}^{n} f(c_i)\Delta x$$

where $\Delta x = \frac{b-a}{n}$ and $c_i = a + i\Delta x$.

Theorem

(Fundamental Theorem of Calculus, Part 1) If f is continuous on $[a, b]$, then the function g defined by

$$g(x) = \int_a^x f(t)dt \quad a \leq x \leq b$$

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Theorem

(Fundamental Theorem of Calculus, Part 2) If f is continuous on $[a, b]$, then

$$\int_a^b f(x)dx = F(b) - F(a)$$

Where F is any antiderivative of f, that is, a function such that $F' = f$.

U-Substitution for definite integrals

Theorem

If $u = g(x)$ is a differentiable function and f is continuous, then

$$\int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u)du$$

Finding the Area Enclosed by Curves

Steps to Find the Area Enclosed by Curves

1. Draw a picture illustrating the inclosed region.
2. Find the points of intersection for all pairs of curves.
3. Decide if you will integrate with respect to x or y.
4. Write down the integral (or sum of integrals) that represents the area and evaluate it.

Good Luck on the Exam!