Math 103: The Substitution Method and the area between curves

Ryan Blair

University of Pennsylvania

Thursday October 1, 2011

Outline

U-Substitution for definite integrals

Theorem

If $u = g(x)$ is a differentiable function and f is continuous, then

$$\int f(g(x))g'(x)dx = \int f(u)du$$

U-Substitution for definite integrals

Theorem

If $u = g(x)$ is a differentiable function and f is continuous, then

$$\int f(g(x))g'(x)dx = \int f(u)du$$

Theorem

If $u = g(x)$ is a differentiable function and f is continuous, then

$$\int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u)du$$

Definite integrals of even and odd functions

Theorem

Let f be a continuous function on the interval [-a,a].

1. *If f is even, then* $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$
2. *If f is odd, then* $\int_{-a}^{a} f(x)dx = 0$

Area Between Curves

Theorem

If f and g are continuous functions with $f(x) \geq g(x)$ throughout $[a, b]$, then the **area of the region between the curves** $y = f(x)$ **and** $y = g(x)$ **from a to b** *is given by*

$$A = \int_a^b [f(x) - g(x)]dx$$

Finding the Area Enclosed by Curves

Steps to Find the Area Enclosed by Curves

1. Draw a picture illustrating the inclosed region.
2. Find the points of intersection for all pairs of curves.
3. Decide if you will integrate with respect to x or y.
4. Write down the integral (or sum of integrals) that represents the area and evaluate it.