Math 103: Optimization

Ryan Blair

University of Pennsylvania

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Outline

1. Midterm Two Info
2. Optimization

Where to Find More Practice Problems for Midterm 2

1. Practice Midterm 2
   http://www.math.upenn.edu/∼ryblair/Math103F11/index.html
2. Old Practice Midterm 2
   http://www.math.upenn.edu/∼ryblair/Math 103/index.html
3. Examples done in class
4. Old Final exam problems
   http://www.math.upenn.edu/ugrad/calc/m103/oldexams.html
5. Homework

Proofs that could be on the exam

1. Use Rolle's theorem to prove the Mean Value Theorem. Page 231.
2. Derive the formula for $\frac{d}{dx}(f^{-1}(x))$. Page 177
3. Derive the formula for $\frac{d}{dx}(sin^{-1}(x))$. Page 188
4. Derive the formula for $\frac{d}{dx}(tan^{-1}(x))$. Page 188
5. Use the Mean value theorem to show that if $f(x)$ and $g(x)$ are everywhere differentiable functions such that $f'(x) = g'(x)$, then there exists a constant C such that $f(x) = g(x) + C$. Page 233.
6. The first derivative theorem for local extreme values. Page 225.

Example

A farmer has 2400ft of fencing and wants to fence off a rectangular field that boarders a straight river. He needs no fence along the river. What are the dimensions of the field that has the largest area?

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Steps to Solving Optimization Problems

1. Draw a picture representing the problem.
2. Introduce variables and find a formula for the quantity being optimized.
3. Use the information in the problem to express the quantity being optimized in terms of a single variable.
4. Use the first derivative test to find the local minima and maxima.
5. Finish solving the problem.

Example

A cylindrical can is to be made to hold 1 L of oil. Find the dimensions that will minimize the cost of the metal to manufacture the can.

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Example

Find the point on the parabola $y^2 = 2x$ that is closest to the point $(1, 4)$.

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Example

Find the dimensions of a rectangle of largest area that can be inscribed in an equilateral triangle of side length L if one side of the rectangle lies on the base of the triangle.

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