Math 103: L'Hopital's Rule

Ryan Blair

University of Pennsylvania

Thursday November 3, 2011

Outline

1. L'Hospital's Rule

2. Review

Indeterminant forms

For some limits evaluation via substation gives meaningless expressions called **Indeterminant Forms**

$$lim_{x \to 0} \frac{sin(x)}{x} = \frac{0}{0}$$

Indeterminant forms

For some limits evaluation via substation gives meaningless expressions called **Indeterminant Forms**

$$lim_{x\to 0}\frac{sin(x)}{x} = \frac{0}{0}$$

$$lim_{x\to\infty}\frac{x}{e^x} = \frac{\infty}{\infty}$$

Indeterminant forms

For some limits evaluation via substation gives meaningless expressions called **Indeterminant Forms**

$$lim_{x \to 0} \frac{sin(x)}{x} = \frac{0}{0}$$

$$lim_{x \to \infty} \frac{x}{e^x} = \frac{\infty}{\infty}$$

$$lim_{x \to \frac{\pi}{2}^-} \ tan(x) - \frac{1}{(x - \frac{\pi}{2})^2} = \infty - \infty$$

Indeterminant forms

For some limits evaluation via substation gives meaningless expressions called **Indeterminant Forms**

$$lim_{x\to 0}\frac{sin(x)}{x} = \frac{0}{0}$$

$$lim_{x\to\infty}\frac{x}{e^x} = \frac{\infty}{\infty}$$

$$lim_{x\to\frac{\pi}{2}^-} \; tan(x) - \frac{1}{(x-\frac{\pi}{2})^2} = \infty - \infty$$

Other indeterminant forms include $\infty \cdot 0$, 0^0 and 1^∞

L'Hospital's Rule for $\frac{0}{0}$

Theorem

Suppose $f(a) = g(a) = 0$, f and g are differentiable near a and $g'(x) \neq 0$ for x near a but not equal to a, Then

$$lim_{x\rightarrow a}\frac{f(x)}{g(x)} = lim_{x\rightarrow a}\frac{f'(x)}{g'(x)}$$

if the right-hand limit exists.

L'Hospital's Rule for $\frac{0}{0}$

Theorem

Suppose $f(a) = g(a) = 0$, f and g are differentiable near a and $g'(x) \neq 0$ for x near a but not equal to a, Then

$$lim_{x \to a} \frac{f(x)}{g(x)} = lim_{x \to a} \frac{f'(x)}{g'(x)}$$

if the right-hand limit exists.

The theorem also holds for one-sided limits and infinite limits.

L'Hospital's Rule for $\frac{0}{0}$

Theorem

Suppose $f(a) = g(a) = 0$, f and g are differentiable near a and $g'(x) \neq 0$ for x near a but not equal to a, Then

$$lim_{x \to a} \frac{f(x)}{g(x)} = lim_{x \to a} \frac{f'(x)}{g'(x)}$$

if the right-hand limit exists.

The theorem also holds for one-sided limits and infinite limits.

This only helps us with indeterminant forms $\frac{0}{0}$.

L'Hospital's Rule for $\frac{\infty}{\infty}$

Theorem

Suppose $f(x) \to \infty$ and $g(x) \to \infty$ as $x \to a$, then

$$lim_{x \to a} \frac{f(x)}{g(x)} = lim_{x \to a} \frac{f'(x)}{g'(x)}$$

if the right-hand limit exists.

L'Hospital's Rule for $\frac{\infty}{\infty}$

Theorem

Suppose $f(x) \to \infty$ and $g(x) \to \infty$ as $x \to a$, then

$$lim_{x \to a} \frac{f(x)}{g(x)} = lim_{x \to a} \frac{f'(x)}{g'(x)}$$

if the right-hand limit exists.

The theorem also holds for one-sided limits and infinite limits.

L'Hospital's Rule for $\frac{\infty}{\infty}$

Theorem

Suppose $f(x) \to \infty$ and $g(x) \to \infty$ as $x \to a$, then

$$lim_{x \to a} \frac{f(x)}{g(x)} = lim_{x \to a} \frac{f'(x)}{g'(x)}$$

if the right-hand limit exists.

The theorem also holds for one-sided limits and infinite limits.

Must convert other indeterminant forms to $\frac{0}{0}$ or $\frac{\infty}{\infty}$.

Guidelines for Curve Sketching

To sketch the graph of $y = f(x)$,

Guidelines for Curve Sketching

To sketch the graph of $y = f(x)$,

1. Find the domain of $f(x)$ and any symmetries.

Guidelines for Curve Sketching

To sketch the graph of $y = f(x)$,

1. Find the domain of $f(x)$ and any symmetries.
2. Find $f'(x)$ and $f''(x)$.

Guidelines for Curve Sketching

To sketch the graph of $y = f(x)$,

1. Find the domain of $f(x)$ and any symmetries.
2. Find $f'(x)$ and $f''(x)$.
3. Find the critical points of f and determine the behavior at each.

Guidelines for Curve Sketching

To sketch the graph of $y = f(x)$,

1. Find the domain of $f(x)$ and any symmetries.
2. Find $f'(x)$ and $f''(x)$.
3. Find the critical points of f and determine the behavior at each.
4. Find where the graph of f is increasing and decreasing.

Guidelines for Curve Sketching

To sketch the graph of $y = f(x)$,

1. Find the domain of $f(x)$ and any symmetries.
2. Find $f'(x)$ and $f''(x)$.
3. Find the critical points of f and determine the behavior at each.
4. Find where the graph of f is increasing and decreasing.
5. Find the points of inflection and the concavity of f.

Guidelines for Curve Sketching

To sketch the graph of $y = f(x)$,

1. Find the domain of $f(x)$ and any symmetries.
2. Find $f'(x)$ and $f''(x)$.
3. Find the critical points of f and determine the behavior at each.
4. Find where the graph of f is increasing and decreasing.
5. Find the points of inflection and the concavity of f.
6. Identify any asymptotes.

Guidelines for Curve Sketching

To sketch the graph of $y = f(x)$,

1. Find the domain of $f(x)$ and any symmetries.
2. Find $f'(x)$ and $f''(x)$.
3. Find the critical points of f and determine the behavior at each.
4. Find where the graph of f is increasing and decreasing.
5. Find the points of inflection and the concavity of f.
6. Identify any asymptotes.
7. Plot key points and asymptotes, and sketch the curve.