Math 103: Indefinite Integrals and the Substitution Method

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Outline

1. Review
2. Indefinite Integral
3. Substitution

Three ways to evaluate a definite integral

1. The area under the curve.
2. The limit definition.
3. The Fundamental Theorem of Calculus.

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Theorem

If f is integrable on $[a, b]$, then

$$\int_a^b f(x)dx = lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\Delta x$$

where $\Delta x = \frac{b-a}{n}$ and the c_i are a collection of sample points.

Three ways to evaluate a definite integral

1. The area under the curve.
2. The limit definition.
3. The Fundamental Theorem of Calculus.

Theorem

(Fundamental Theorem of Calculus, Part 2) If f is continuous on $[a, b]$, then

$$\int_a^b f(x)dx = F(b) - F(a)$$

Where F is any antiderivative of f, that is, a function such that $F' = f$.

Indefinite Integral

Definition

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Key Idea: To find the indefinite integral of functions we don't know the antiderivative of we can run chain rule backwards.

The Substitution Rule

Theorem

If $u = g(x)$ is a differentiable function and f is continuous, then

$$\int f(g(x))g'(x)dx = \int f(u)du$$