Math 103: Concavity and Using Derivatives to Graph a Function

Ryan Blair

University of Pennsylvania

Tuesday November 1, 2011

Outline

First Derivative Test

Suppose that c is a critical number of a continuous function f.

First Derivative Test

Suppose that c is a critical number of a continuous function f.

1. If f' changes from positive to negative at c, then f has a local maximum at c.

First Derivative Test

Suppose that c is a critical number of a continuous function f.

1. If f' changes from positive to negative at c, then f has a local maximum at c.
2. If f' changes from negative to positive at c, then f has a local minimum at c.

First Derivative Test

Suppose that c is a critical number of a continuous function f.

1. If f' changes from positive to negative at c, then f has a local maximum at c.
2. If f' changes from negative to positive at c, then f has a local minimum at c.
3. If f does not change sign at c, then f has no local maximum or minimum at c.

Definition

If a graph of f lies above all of its tangents on an interval I, then is is called **concave up** on I. If a graph of f lies below all of its tangents on an interval I, then is is called **concave down** on I.

Definition

If a graph of f lies above all of its tangents on an interval I, then is is called **concave up** on I. If a graph of f lies below all of its tangents on an interval I, then is is called **concave down** on I.

Concavity test

1. If $f"(x) > 0$ for all x in I, then the graph of f is concave up on I.
2. If $f"(x) < 0$ for all x in I, then the graph of f is concave down on I.

Definition

A point P on a continuous curve $y = f(x)$ is called and **inflection point** if f changes from concave down to concave up or visa versa at P.

The Second Derivative Test

Suppose f" is continuous near c.

1. If $f'(c) = 0$ and $f"(c) > 0$, then f has a local minimum at c.
2. If $f'(c) = 0$ and $f"(c) < 0$, then f has a local maximum at c.

Guidelines for Curve Sketching

To sketch the graph of $y = f(x)$,

Guidelines for Curve Sketching

To sketch the graph of $y = f(x)$,

1. Find the domain of $f(x)$ and any symmetries.

Guidelines for Curve Sketching

To sketch the graph of $y = f(x)$,

1. Find the domain of $f(x)$ and any symmetries.
2. Find $f'(x)$ and $f''(x)$.

Guidelines for Curve Sketching

To sketch the graph of $y = f(x)$,

1. Find the domain of $f(x)$ and any symmetries.
2. Find $f'(x)$ and $f''(x)$.
3. Find the critical points of f and determine the behavior at each.

Guidelines for Curve Sketching

To sketch the graph of $y = f(x)$,

1. Find the domain of $f(x)$ and any symmetries.
2. Find $f'(x)$ and $f''(x)$.
3. Find the critical points of f and determine the behavior at each.
4. Find where the graph of f is increasing and decreasing.

Guidelines for Curve Sketching

To sketch the graph of $y = f(x)$,

1. Find the domain of $f(x)$ and any symmetries.
2. Find $f'(x)$ and $f''(x)$.
3. Find the critical points of f and determine the behavior at each.
4. Find where the graph of f is increasing and decreasing.
5. Find the points of inflection and the concavity of f.

Guidelines for Curve Sketching

To sketch the graph of $y = f(x)$,

1. Find the domain of $f(x)$ and any symmetries.
2. Find $f'(x)$ and $f''(x)$.
3. Find the critical points of f and determine the behavior at each.
4. Find where the graph of f is increasing and decreasing.
5. Find the points of inflection and the concavity of f.
6. Identify any asymptotes.

Guidelines for Curve Sketching

To sketch the graph of $y = f(x)$,

1. Find the domain of $f(x)$ and any symmetries.
2. Find $f'(x)$ and $f''(x)$.
3. Find the critical points of f and determine the behavior at each.
4. Find where the graph of f is increasing and decreasing.
5. Find the points of inflection and the concavity of f.
6. Identify any asymptotes.
7. Plot key points and asymptotes, and sketch the curve.

Definition

The line $y = mx + b$ is a slant asymptote for $f(x)$ if

$$lim_{x\to\infty}[f(x) - (mx + b)] = 0$$

If $f(x) = \frac{p(x)}{q(x)}$ where $q(x)$ and $p(x)$ are polynomials, then $f(x)$ has a **slant asymptote** if and only if the degree of $p(x)$ is one more than the degree of $q(x)$.