Math 103: The Mean Value Theorem and How Derivatives Shape a Graph

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Outline

Review

Last time we learned

1. How to find local minima and maxima.
2. How to find absolute minima and maxima.
3. How the derivative relates to minima and maxima.

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Theorem

Suppose $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b). If $f(a) = f(b)$, then there exists a number c such that $a < c < b$ and $f'(c) = 0$.

The Mean Value Theorem

Theorem

Suppose $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b). Then there exists a number c such that $a < c < b$ and

$$\frac{f(b) - f(a)}{b - a} = f'(c).$$

Important Consequence

Theorem

If $f'(x) = g'(x)$ for all points in (a, b), then there exists a constant C such that $f(x) = g(x) + C$ for all points in (a, b).

Increasing and Decreasing

Recall from last time

Theorem

(Fermat's Theorem)
If f has a local maximum or minimum at c, and if $f'(c)$ exists, then $f'(c) = 0$.

But we can say more:

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Theorem

Suppose f is differentiable on $[a, b]$.

1. *If $f'(x) > 0$ on $[a, b]$, then f is increasing on $[a, b]$.*
2. *If $f'(x) < 0$ on $[a, b]$, then f is decreasing on $[a, b]$.*

First Derivative Test

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Suppose that c is a critical number of a continuous function f.

1. If f' changes from positive to negative at c, then f has a local maximum at c.
2. If f' changes from negative to positive at c, then f has a local minimum at c.
3. If f does not change sign at c, then f has no local maximum or minimum at c.

Definition

If a graph of f lies above all of its tangents on an interval I, then is is called **concave up** on I. If a graph of f lies below all of its tangents on an interval I, then is is called **concave down** on I.

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Concavity test

1. If $f''(x) > 0$ for all x in I, then the graph of f is concave up on I.
2. If $f''(x) < 0$ for all x in I, then the graph of f is concave down on I.

Definition

A point P on a continuous curve $y = f(x)$ is called and **inflection point** if f changes from concave down to concave up or visa versa at P.

The Second Derivative Test

Suppose f'' is continuous near c.

1. If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at c.
2. If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at c.

Example $Let f(x) = 3x^{\frac{2}{3}} - x$

1. Find intervals of increase and decrease
2. find all local max and min
3. find intervals of concavity and the inflection points
4. Sketch the graph