Math 103: Extreme Values of Functions and the Mean Value Theorem

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Tuesday October 25, 2011

Outline

Definition

A function f has an **absolute maximum** at c if $f(c) \geq f(x)$ for all x in the domain of f. $f(c)$ is the **maximum value** of f.

A function f has an **absolute minimum** at c if $f(c) \leq f(x)$ for all x in the domain of f. $f(c)$ is the **minimum value** of f

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A function f has an **local maximum** at c if $f(c) \geq f(x)$ when x is near c.

A function f has an **local minimum** at c if $f(c) \leq f(x)$ when x is near c.

Theorem

(Extreme Value Theorem)
If f is continuous on a closed interval $[a, b]$, then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$.

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(Fermat's Theorem)
If f has a local maximum or minimum at c, and if $f'(c)$ exists, then $f'(c) = 0$.

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A **Critical Point** of a function f is a number c in the domain of f such that either $f'(c) = 0$ or $f'(c)$ is undefined.

The Closed Interval Method

To find the absolute maximum and minimum values of a continuous function f on a closed interval $[a, b]$:

Step 1: Find the values of f at the critical numbers of f in (a, b).

Step 2: Find the values of f at a and b.

Step 3: The largest of the values from step 1 and step 2 is the absolute maximum value; the smallest of the values from step 1 and step 2 is the absolute minimum value.

Rolle's Theorem

Theorem

Suppose $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b). If $f(a) = f(b)$, then there exists a number c such that $a < c < b$ and $f'(c) = 0$.

The Mean Value Theorem

Theorem

Suppose $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b). Then there exists a number c such that $a < c < b$ and

$$\frac{f(b) - f(a)}{b - a} = f'(c).$$

Important Consequence

Theorem

If $f'(x) = g'(x)$ for all points in (a, b), then there exists a constant C such that $f(x) = g(x) + C$ for all points in (a, b).