### Math 103: Derivatives of Inverse Functions and Logs

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Suppose f(x) is a function with inverse  $f^{-1}(x)$  with each defined on the appropriate domain and range.

$$\frac{d}{dx}(f^{-1}(x)) = \frac{1}{f'(f^{-1}(x))}$$

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**Exercise** Find the slope of the tangent line to  $y = x^2$  at (2, 4) and find the slope of the tangent line to  $y = \sqrt{(x)}$  at (4, 2).

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Derivatives of Inverse Functions and Logs

### Properties of Logarithmic Functions

# • If a > 1 and x, y > 0, then $log_a(xy) = log_a(x) + log_a(y)$ .

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 and  $x, y > 0$ , then  
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 $log_a(\frac{x}{y}) = log_a(x) - log_a(y)$ .

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### Change of Base Formula

## For any positive number $a \ (a \neq 0)$ , we have

$$log_a(x) = rac{ln(x)}{ln(a)}$$

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Image: A matrix and a matrix

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#### Derivatives of Inverse Trig Functions

$$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$$

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•  $\frac{d}{dx}(cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}}$ 

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 $\frac{d}{dx}(csc^{-1}(x)) = \frac{-1}{|x|\sqrt{x^2-1}}$