MATH 103 - Fall 2011
Practice Midterm Two

Name:

TA:

Recitation Number:

You may use both sides of a 8.5×11 sheet of paper for notes while you take this exam. No calculators, no course notes, no books, no help from your neighbors. Show all work, even on multiple choice or short answer questions—I will be grading as much on the basis of work shown as on the end result. Include all units. Remember to put your name at the top of this page. Good luck.

Problem	Score (out of)
1	(10)
2	(10)
3	(10)
4	(10)
5	(10)
6	(10)
7	(10)
8	(10)
9	(10)
Total	(90)

1. (10 points) Find the derivative of the following function.

$$f(x) = ln|tan^{-1}(\frac{e^x}{x})|$$

2.(10 points) Use the Mean Value Theorem to prove the following theorem

If $f(x)$ and $g(x)$ are everywhere differentiable functions such that $f'(x) = g'(x)$, then there exists a constant C such that $f(x) = g(x) + C$.

3.(10 points) A continuous and differentiable function f decreases steadily on the interval $[-4, 1]$, and satisfies $f(-4) = 1$ and $f(1) = -1$. Which of the following **CANNOT** be true? Circle **ALL** that apply, and justify your answers.

(A) f has a local maximum at some c in $(-4, 1)$.

(B) f has an infection point at some c in $(-4, 1)$.

(C) $f'(x) \leq -2$ for all x in $[-4, 1]$.

(D) $f'(x) \geq -1$ for all x in $[-4, 1]$.

4.(10 points) Let $f(x) = 3x^4 + 4x^3 - 12x^2 + 3$. How many zeros does the function f have in the interval $[1, 3]$? For credit you must justify your answer. **Hint:** Where is $f(x)$ increasing?

5. (10 points) Find the critical points and intervals of concavity of $f(x) = x + cos(x)$ on the interval $[-2\pi, 2\pi]$. What does the second derivative test tell you about the critical points?

6. (10 points) Find the following limit

$$lim_{x \to \infty}(1 + \frac{1}{x})^x$$

7. (10 points) **Carefully** draw a graph for a function $f(x)$ given the following data. Be sure to label all maxima and minima and all asymptotes.

A) The domain of $f(x)$ is $(-\infty, 0) \cup (1, \infty)$.

B) $f(x)$ has no x-intercepts.

C) The interval of increase of $f(x)$ is $(-\infty, -2)$.

D) The interval of decrease of $f(x)$ is $(-2, 0) \cup (1, \infty)$.

E) $f(x)$ has a critical value at $x = -2$.

F) $f(x)$ has vertical asymptotes at $x = 0$ and $x = 1$.

G) $f(x)$ has horizontal asymptotes $y = 1$ and $y = -1$.

8. (10 points) Suppose that a drop of mist is a perfect sphere and that, through condensation, the drop picks up moisture at a rate proportional to its surface area. Show that under these circumstances the drop's radius increases at a constant rate.

9. (10 points) A right circular cylinder is inscribed in a sphere of radius 3 cm. Find the largest possible volume of such a cylinder.