

1. PRACTICE MIDTERM 1

Problem 1. At what value(s) of x is the following function discontinuous?

$$f(x) = \begin{cases} x^2 + 4x + 5 & : \text{if } x < -2 \\ \frac{1}{2}x & : \text{if } -2 \leq x \leq 2 \\ 1 + \sqrt{x-2} & : \text{if } x > 2 \end{cases}$$

Problem 2. Find the derivative of $f(x) = \sqrt{x^2 + 1}$ using the **limit definition** of derivative.

Problem 3. Prove the following product rule for derivatives

$$\frac{d}{dx}(f(x)g(x)) = f(x)\frac{d}{dx}(g(x)) + g(x)\frac{d}{dx}(f(x))$$

Problem 4. What is the slope of the tangent line to $f(x) = (x^2)(e^x)$ at $x = 2$?

Problem 5. Find the value of the limit.

$$\lim_{x \rightarrow 2} \frac{\sqrt{x+7} - 3}{(x-2)(x+1)}$$

Problem 6. Find the value of the limit.

$$\lim_{x \rightarrow \infty} \frac{\sqrt{2x^2 + 1}}{1 - 3x}$$

Problem 7. Find values a and b such that $f(x)$ is differentiable everywhere

$$f(x) = \begin{cases} x^{\frac{1}{2}} + x & : \text{if } x \geq 1 \\ ax^2 + bx + 1 & : \text{if } x < 1 \end{cases}$$

Problem 8. Find all points on the graph of $f(x) = x^3 - 2x$ where the tangent line has slope 1.

Problem 9. Evaluate the following limit.

$$\lim_{x \rightarrow 0} \frac{x \csc(2x)}{\cos(5x)}$$