1. Homework 2

Due: In Lecture 9-21

Problem 1. Show that if $A \subset \mathbb{R}^n$ is a rectangle and $f : A \to \mathbb{R}$ is continuous, then f is Riemann integrable on A.

Problem 2. From definitions show that if f and g are Riemann integrable on $A \subset \mathbb{R}^n$ a rectangle, then so is $f + g$, and

$$\int_A (f+g) = \int_A f + \int_A g,$$

Problem 3. Show that if f and g are Riemann integrable on $A \subset \mathbb{R}^n$ a rectangle, then so is $f * g$. Feel free to use theorems presented in lecture.

Problem 4. Show that a compact set $A \subset \mathbb{R}^n$ has measure zero if and only if it has content zero.

Problem 5. Let A be a closed subset of $\mathbb{R}^n$ and $f : A \to \mathbb{R}$ a bounded function. Show that the set

$$\{x \epsilon A : o(f,x) \geq \varepsilon\}$$

is closed for any $\varepsilon > 0$.

Problem 6. Let C be a bounded set in $\mathbb{R}^n$ and $A \subset \mathbb{R}^n$ be a rectangle such that $C \subset A$. Show that χ_C is Riemann integrable on A if and only if the boundary of C has measure zero.