

1. HOMEWORK 11

Due: In Lecture 11-2

Problem 1. Define $f : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ by

$$f(u, v) = (x(u, v), y(u, v)) = (u^2 - v^2, 2uv).$$

Let $\omega = -y \, dx + x \, dy$, and compute $f^*\omega$.

Comment. Pulling forms back is mechanical, and feels like substitution.

Problem 2. If ω and η are differential k - and r -forms, then

$$d(\omega \wedge \eta) = d\omega \wedge \eta + (-1)^k \omega \wedge d\eta$$

Problem 3.(a) Compute the exterior derivative of each of the following forms:

- (1) $x \, dy + y \, dx$
- (2) $x \, dy - y \, dx$
- (3) $x \, dx \, dy$
- (4) $z \, dx \, dy + y \, dx \, dz$

(b) In each case that the exterior derivative is zero, find a form of one degree less whose exterior derivative is the given form.