1. Practice Midterm 1

Problem 1. At what value(s) of x is the following function discontinuous?

$$f(x) = \begin{cases} x^2 + 4x + 5 & : if \;\; x < -2 \\ \frac{1}{2}x & : if \;\; -2 \leq x \leq 2 \\ 1 + \sqrt{x-2} & : if \;\; x > 2 \end{cases}$$

a) -2
b) 0
c) -2,0, and 2
d) -2 and 0
e) 2
f) -2 and 2
g) 0 and 2
h) f is continuous everywhere

Problem 2. The hypotenuse AB of a right triangle ABC remains constant at 5 feet as both legs are changing. One leg, AC, is decreasing at the rate of 2 feet per second. In order for the hypotenuse to remain 5 feet, the other leg, BC, is increasing. The rate, in square feet per second, at which the **area** is changing when $AC = 3$ is

a) $\frac{25}{4}$
b) $\frac{7}{2}$
c) $\frac{-3}{2}$
d) $\frac{-7}{4}$
e) $\frac{3}{2}$
f) $\frac{-7}{2}$
g) $\frac{7}{4}$
h) None of these

Problem 3

If $x^2 - xy - y^3 = 13$, then find $\frac{dy}{dx}$ evaluated at $(4, 1)$.

a) 0
b) $\frac{3}{2}$
c) $\frac{7}{2}$
d) $\frac{9}{7}$
e) -2
f) -1
g) 1
h) 7

Problem 4. What is the slope of the tangent line to $f(x) = (x)(cos(x^2))$ at $x = \sqrt{\frac{\pi}{2}}$?

a) $-\pi$
b) π
c) 0
d) 1
e) -1
f) $\frac{1}{2}$

Problem 5. The function $f(x) = (x-3)^{\frac{2}{3}}$ is increasing for what values of x?

a) $(-\infty, \infty)$
b) $(3, \infty)$
c) nowhere
d) $(-\infty, 3)$
e) $(0, \infty)$
f) everywhere except 3

Problem 6.

Use the intermediate value theorem to show that there is a number that is exactly one more than its cube.

Problem 7. Find the value of the limit.

$$lim_{x\to 2}\frac{\sqrt{x+7}-3}{(x-2)(x+1)}$$

Problem 8. Let V be the volume of a cylinder having height h and radius r, and assume that h and r vary with time. When the height is 5 in. and is increasing at 0.2 in./s. , the radius is 3 in. and is decreasing at 0.1 in./s. How fast is the volume changing at that instant?

Problem 9. Suppose $f(3) = 2$, $f'(3) = 5$, and $f"(3) = -2$. Let $g(x) = [f(x)]^2$. Find the value of $g"(3)$.

Problem 10. If $f(x) = \frac{x}{tan(x)}$, find $f'(\frac{\pi}{4})$. Do not leave any trigonometric functions in your answer.