Math 103 Day 8: Implicit Differentiation and Rates of Change

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Outline

1 Implicit Differentiation

Most of the functions we have investigated so far can be described by expressing one variable in terms of another explicitly.

1. $y = x^2 + 2$
2. $y = sin(x)$
3. $y = \sqrt{(sin(x))^2 + 1}$

However, some functions are better defined implicitly.

1. $x^2 + y^2 = 1$
2. $y^5 + 3x^2y^2 + 5x^4 = 12$
3. $2(x^2 + y^2)^2 = 25(x^2 - y^2)$
4. $cos(x)sin(y) = 1$

Goal: Find y' without having to solve for y.

To find the derivative of an implicitly defined function, the key is to remember **y is a function of x** and to use the **chain rule**.

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1. $v(t) = \frac{ds}{dt}$ represents the instantaneous velocity
2. $a(t) = \frac{dv}{dt} = s''(t)$ represents the instantaneous acceleration

ExampleThe position of a particle is given by the equation

$$s(t) = t^3 + 3t^2 + 2t$$

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1. Find the velocity at time t.
2. When is the particle at rest?
3. When is the particle moving forward?
4. Find the acceleration at time t.
5. Graph the position, velocity and acceleration functions.
6. When is the particle speeding up?

Example

A spherical balloon is being inflated. Find the rate of increase of the surface area (in ft^2) with respect to the radius r when r is $1ft$, $2ft$, $3ft$ and for arbitrary r. What conclusion can you make?

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Example

The cost in dollars of producing x yards of a certain fabric is

$$C(x) = 1200 + 12x - 0.1x^2 + .0005x^3.$$

Find the marginal cost function.

Find $C'(200)$ and explain its meaning. What does it predict?

Example

Newton's Law of Gravitation says that the magnitude F of the force exerted by a body of mass m on a body of mass M is

$$F = \frac{GmM}{r^2}$$

where G is the gravitational constant and r is the distance between the bodies.

Find $\frac{dF}{dr}$ and explain its meaning.

Suppose it is known that the earth attracts an object with a force that decreases at the rate of $2\frac{N}{km}$ when $r = 20000$. How fast does the force change when $r = 10000$?