Math 103 Day 5: Derivatives

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Outline

Tangent Lines

Definition

The tangent line to a curve $y = f(x)$ at a point $(a, f(a))$ is the line through $(a, f(a))$ with the slope

$$lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

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Definition

(Alternative)The slope of the tangent line at $(a, f(a))$ is given by

$$lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Definition

If $s(t)$ is a position function defined in terms of time t, then the instantaneous velocity at time $t = a$ is given by

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ExampleSuppose a penny is dropped from the top of DRL which is 19.6 meters high. The position of the penny in terms of hight above the street is given by $s(t) = 19.6 - 4.9t^2$. At what speed is the penny traveling when it hits the ground.

Derivative

Definition

The derivative of a function f at a number a, denoted by $f'(a)$, is

$$f'(a) = lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

If the limit exists.

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Note. Another name for the derivative of f at a is the **instantaneous rate of change** of f at a.

Derivative as a function

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Notation. Other ways of writing the derivative of $y = f(x)$.

$$f'(x) = y' = \frac{dy}{dx} = \frac{df}{dx} = \frac{d}{dx}f(x) = Df(x) = D_x f(x)$$

Theorem

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However, using our limit laws, this is equivalent to showing

$$lim_{x \to a}(f(x) - f(a)) = 0.$$

Theorem

If f is differentiable at a, then f is continuous at a.

To prove the theorem we will assume

$$f'(a) = lim_{x \longrightarrow a} \frac{f(x) - f(a)}{x - a}$$

and we will show

$$lim_{x \longrightarrow a}(f(x) - f(a)) = 0.$$

Higher Derivatives

If $y = f^{I}(x)$, then $\frac{dy}{dx} = f^{II}(x)$

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In general, the "n-th" derivative of $f(x)$ is denoted by $f^{(n)}(x)$.