# Math 103 Day 4: Continuity

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# Outline

1 Continuity

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## Definition

A function $f$ is continuous at a number $a$ if

$$lim_{x \to a} f(x) = f(a)$$

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## Definition

(More Precisely)$f$ is continuous at $a$ if

1. $f(a)$ is defined (in the domain)
2. $lim_{x \to a} f(x)$ exists
3. $lim_{x \to a} f(x) = f(a)$

# More Continuity

## Definition

A function $f$ is continuous from the right at a number $a$ if

$$lim_{x \to a^+} f(x) = f(a)$$

## Definition

A function $f$ is continuous from the left at a number $a$ if

$$lim_{x \to a^-} f(x) = f(a)$$

# More Continuity

### Definition

A function $f$ is continuous from the right at a number $a$ if

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$$lim_{x \to a^-} f(x) = f(a)$$

### Definition

A function $F$ is continuous on an interval if it is continuous at every number in the interval. (At the endpoints, if there are any, we define continuity as continuity from the left or the right.)

## Theorem

*If $f$ and $g$ are continuous at $a$ and $c$ is a constant, then the following functions are continuous at $a$:*

1. $f + g$
2. $f - g$
3. $cf$
4. $f \times g$
5. $\frac{f}{g}$ *if* $g(a) \neq 0$

## Theorem

*The following types of functions are continuous at every point in their domains:*

1. *polynomials*
2. *rational functions*
3. *root functions*
4. *trigonometric functions*

**Theorem**

*If $f$ is continuous at $b$ and $lim_{x \to a} g(x) = b$, then*

$$lim_{x \to a} f(g(x)) = f(lim_{x \to a} g(x)).$$

**Theorem**

*If $g$ is continuous at $a$ and $f$ is continuous at $g(a)$, then $f \circ g(x) = f(g(x))$ is continuous at $a$.*

### Theorem

*(Intermediate Value Theorem) Suppose $f$ is continuous on the closed interval $[a, b]$ and let $N$ be any number between $f(a)$ and $f(b)$ where $f(a) \neq f(b)$. Then there exists a number $c$ in $(a, b)$ s.t. $f(c) = N$.*

# The Monk Riddle

A monk leaves the monastery at 7am and takes his usual path to the top of the mountain, arriving at 7pm. The following morning, he starts at the top and takes the same path back, arriving at the monastery at 7pm. Show there is a point on the path that the monk will cross at exactly the same time of day on both days.