Math 103 Day 3: More Limits

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Outline

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Infinite Limits

Definition

Let f be a function defined on both sides of a, except possibly at a itself, then

$$lim_{x \longrightarrow a} f(x) = \infty$$

if $f(x)$ can be made arbitrarily large by taking x sufficiently close to a, but not equal to a

Infinite Limits

Definition

Let f be a function defined on both sides of a, except possibly at a itself, then

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if $f(x)$ can be made arbitrarily large by taking x sufficiently close to a, but not equal to a

Definition

Let f be a function defined on both sides of a, except possibly at a itself, then

$$lim_{x \to a} f(x) = -\infty$$

if $f(x)$ can be made arbitrarily large and negative by taking x sufficiently close to a, but not equal to a

Vertical Asymptotes

Definition

The line $x = a$ is called a vertical asymptote of $y = f(x)$ if one of the following holds:

1. $lim_{x \to a^-} = \infty$
2. $lim_{x \to a^-} = -\infty$
3. $lim_{x \to a^+} = \infty$
4. $lim_{x \to a^+} = -\infty$

Limit Laws I

1. $lim_{x \to a}[f(x) + g(x)] = lim_{x \to a} f(x) + lim_{x \to a} g(x)$
2. $lim_{x \to a}[f(x) - g(x)] = lim_{x \to a} f(x) - lim_{x \to a} g(x)$
3. $lim_{x \to a}[cf(x)] = c[lim_{x \to a} f(x)]$
4. $lim_{x \to a}[f(x)g(x)] = [lim_{x \to a} f(x)][lim_{x \to a} g(x)]$
5. $lim_{x \to a} \frac{f(x)}{g(x)} = \frac{lim_{x \to a} f(x)}{lim_{x \to a} g(x)}$

Limit Laws II

1. $lim_{x \to a}[f(x)]^n = [lim_{x \to a} f(x)]^n$
2. $lim_{x \to a} c = c$
3. $lim_{x \to a} x = a$
4. $lim_{x \to a} x^{\frac{1}{n}} = a^{\frac{1}{n}}$
5. $lim_{x \to a}[f(x)]^{\frac{1}{n}} = [lim_{x \to a} f(x)]^{\frac{1}{n}}$
6. $lim_{x \to a} sin(f(x)) = sin(lim_{x \to a} f(x))$

1. $lim_{x\to a}[f(x)+g(x)] = lim_{x\to a}f(x) + lim_{x\to a}g(x)$
2. $lim_{x\to a}[f(x)-g(x)] = lim_{x\to a}f(x) - lim_{x\to a}g(x)$
3. $lim_{x\to a}[cf(x)] = c[lim_{x\to a}f(x)]$
4. $lim_{x\to a}[f(x)g(x)] = [lim_{x\to a}f(x)][lim_{x\to a}g(x)]$
5. $lim_{x\to a}\frac{f(x)}{g(x)} = \frac{lim_{x\to a}f(x)}{lim_{x\to a}g(x)}$
6. $lim_{x\to a}[f(x)]^n = [lim_{x\to a}f(x)]^n$
7. $lim_{x\to a}c = c$
8. $lim_{x\to a}x = a$
9. $lim_{x\to a}x^{\frac{1}{n}} = a^{\frac{1}{n}}$
10. $lim_{x\to a}[f(x)]^{\frac{1}{n}} = [lim_{x\to a}f(x)]^{\frac{1}{n}}$
11. $lim_{x\to a}sin(f(x)) = sin(lim_{x\to a}f(x))$

Theorem

If f is a polynomial (or rational function) and a is in the domain of f, then

$$lim_{x \to a} f(x) = f(a).$$

Theorem

If $f(x) \leq g(x)$ when x is near a and the limits of f and g both exist as x approaches a, then

$$lim_{x \to a} f(x) \leq lim_{x \to a} g(x).$$

Theorem

If $f(x) \leq g(x) \leq h(x)$ when x is near a and

$$lim_{x \to a} f(x) = lim_{x \to a} h(x) = L$$

$$\text{then } \; lim_{x \to a} g(x) = L$$