Math 103 Day 16: Optimization

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Outline

Steps to Solving Optimization Problems

1. Draw a picture representing the problem.
2. Find a formula for the quantity being optimized.
3. Use the information in the problem to express the quantity being optimized in terms of a single variable.
4. Use the first derivative test to find the local minima and maxima.
5. Finish solving the problem.

Example

A farmer has 2400ft of fencing and wants to fence off a rectangular field that boarders a straight river. He needs no fence along the river. What are the dimensions of the field that has the largest area?

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Example

A cylindrical can is to be made to hold 1 L of oil. Find the dimensions that will minimize the cost of the metal to manufacture the can.

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3. Use the information in the problem to express the quantity being optimized in terms of a single variable.
4. Use the first derivative test to find the local minima and maxima.
5. Finish solving the problem.

Example

Find the point on the parabola $y^2 = 2x$ that is closest to the point $(1, 4)$.

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4. Use the first derivative test to find the local minima and maxima.
5. Finish solving the problem.

Example

Find the dimensions of a rectangle of largest area that can be inscribed in an equilateral triangle of side length L if one side of the rectangle lies on the base of the triangle.

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4. Use the first derivative test to find the local minima and maxima.
5. Finish solving the problem.