Math 103 Day 12: Maximum and Minimum Values and Linear Approximation

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Outline

We want to be able to find the minima and maxima of functions

Definition

A function f has an **absolute maximum** at c if $f(c) \geq f(x)$ for all x in the domain of f. $f(c)$ is the **maximum value** of f.

A function f has an **absolute minimum** at c if $f(c) \leq f(x)$ for all x in the domain of f. $f(c)$ is the **minimum value** of f

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Definition

A function f has an **local maximum** at c if $f(c) \geq f(x)$ when x is near c.

A function f has an **local minimum** at c if $f(c) \leq f(x)$ when x is near c.

Theorem

(Extreme Value Theorem)
If f is continuous on a closed interval $[a, b]$, then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$.

Theorem

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If f has a local maximum or minimum at c, and if $f'(c)$ exists, then $f'(c) = 0$.

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A **Critical Number** of a function f is a number c in the domain of f such that either $f'(c) = 0$ or $f'(c)$ does not exist.

The Closed Interval Method

To find the absolute maximum and minimum values of a continuous function f on a closed interval $[a, b]$:

Step 1: Find the values of f at the critical numbers of f in (a, b).

Step 2: Find the values of f at the endpoints of the interval.

Step 3: The largest of the values from step 1 and step 2 is the absolute maximum value; the smallest of the values from step 1 and step 2 is the absolute minimum value.

Linear Approximations

The tangent line at $(a, f(a))$ is an approximation of $f(x)$ when x is near a.

The tangent line to $f(x)$ at the point $(a, f(a))$ is given by the formula

$$y = f(a) + f'(a)(x - a)$$

Definition

The linearization of f at a is given by:

$$L(x) = f(a) + f'(a)(x - a)$$