

1. If $A = \begin{bmatrix} 6 & -5 \\ 5 & -4 \end{bmatrix}$ Compute e^{tA} .

answer $e^{tA} = e^{t(I+t(A-I))} = e^t \begin{bmatrix} 1+5t & -5t \\ 5t & 1-5t \end{bmatrix}$

2. If $A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & -1 & 2 \end{bmatrix}$ Solve the equation $\frac{d\vec{x}}{dt}(t) = A\vec{x}(t)$ with $\vec{x}(0) = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

answer $e^{tA} = \begin{bmatrix} e^{2t} & 0 & 0 \\ e^{2t}-e^t & (1-t)e^t & te^t \\ e^{2t}-e^t & -te^t & (1+t)e^t \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} e^{2t} \\ e^{2t}-2(1-t)e^t \\ e^{2t}+2te^t \end{bmatrix}$

3. If $A = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}$ Compute e^{tA}

answer $e^{tA} = \begin{bmatrix} e^t & te^t & \frac{1}{2}t^2e^t & 0 & 0 \\ 0 & e^t & te^t & 0 & 0 \\ 0 & 0 & e^t & 0 & 0 \\ 0 & 0 & 0 & e^{2t} & 2te^{2t} \\ 0 & 0 & 0 & 0 & e^{2t} \end{bmatrix}$ note if $B = \begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}$

$e^{tB} = \begin{bmatrix} e^{2t} & 2te^{2t} \\ 0 & e^{2t} \end{bmatrix}$