1. For each of the following systems of equations indicate if the number of solutions, n = 0 for no solution, n = 1 for a unique solution and n = ∞ for infinitely many solutions. Cirlce your answer.

I	II	III
$x + 2y + 3z = 6$	$x + 2y + 3z = 6$	$x + 2y + 3z = 6$
$2y + 3z = 5$	$2y + 3z = 5$	$2y + 3z = 5$
$x + 4y + 6z = 11$	$x + 4y + 7z = 11$	$x + 4y + 6z = 10$
(0 1 ∞##)	(0 1## ∞)	(0## 1 ∞)

2. Find the point $\vec{r} = (x,y,z)$ on the plane $x + y + z = 3$ that is left invariant by the rotation i.e. $R\vec{r} = \vec{r}$ and $x + y + z = 3$

$$R = \begin{bmatrix} 2/3 & 2/3 & -1/3 \\ 2/3 & -1/3 & 2/3 \\ 1/3 & -2/3 & -2/3 \end{bmatrix}$$

answer $\vec{r} = (2,1,0)$

3. Find the point $\vec{r} = (x,y,z)$ on the plane $x + y + z = 1$ that is left invariant by the rotation i.e. $R\vec{r} = \vec{r}$ and $x + y + z = 1$

$$R = \begin{bmatrix} 2/7 & 3/7 & 6/7 \\ -3/7 & 6/7 & -2/7 \\ -6/7 & -2/7 & 3/7 \end{bmatrix}$$

answer $\vec{r} = (0,2,-1)$

Math 240 (Powers) 80 minute test Thursday Oct. 27, 2016

1. Compute the determinant of the matrix $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & -1 \\ 1 & 1 & 1 & 2 \end{bmatrix}$ Determinant =
(10 points)

A. -1 ## B. 0 C. 1 D. 2 E. 3 F. 4 G. 5 H. none of these

2. Compute the exponential e^{tA} of $A = \begin{bmatrix} 4 & 3 \\ 3 & 4 \end{bmatrix}$. If $e^{tA} = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix}$
(10 points)

Then r_{12} =

A. $\frac{e^{7t}-e^{t}}{2}$ ## B. $\frac{e^{6t}-e^{2t}}{2}$ C. $\frac{e^{7t}-e^{3t}}{2}$ D. $\frac{e^{7t}-e^{4t}}{2}$ E. $\frac{e^{7t}-e^{5t}}{2}$ F. $\frac{e^{7t}-e^{6t}}{2}$ G. $\frac{e^{7t}-e^{7t}}{2}$ H. none of these

3. Compute the exponential e^{tA} of $A = \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}$. If $e^{tA} = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix}$
(10 points)

Then r_{12} =

A. $e^{t}\cos(t)$ B. $e^{2t}\sin(t)$ ## C. $-e^{t}\sin(2t)$ D. $e^{t}\sin(2t)$ E. $-e^{t}\sin(t)$ F. $e^{2t}\sin(2t)$ G. $-e^{2t}\sin(2t)$ H. none of these

4. A (3 x 3)-matrix A has eigenvalues (0,1,2) with corresponding eigenvectors
(15 points)

$\lambda=0 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $\lambda=1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ $\lambda=2 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ Find the sum of the entries of A. $\sum_{i,j=1}^{3} a_{ij}$ =

A. -1 B. 0 C. 2 D. 4 E. 6 ## F. 8 G. 9 H. none of these

20 points

5. If $A = \begin{bmatrix} 1 & -1 & -3 \\ 0 & 0 & -1 \\ 0 & 0 & -1 \end{bmatrix}$ and the exponential $e^{tA} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$. What is r_{13}?

Ans. r_{13} =

A. $2e^{-t}-2$ B. $2e^{-1}+1+e^{t}$ C. $e^{-t}-1$ D. $2e^{-t}-1-e^{t}$ ## E. $2e^{t}-2$ F. $1-e^{-t}$ G. 0 H. none of these

$$e^{tA} = \begin{bmatrix} e^{t} & 1-e^{t} & 2e^{-t}-1-e^{t} \\ 0 & 1 & e^{-t}-1 \\ 0 & 0 & e^{-t} \end{bmatrix}$$

20points

6. If $A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 2 & 0 \end{bmatrix}$ compute the exponential e^{tA} and compute the (1,2) entry.

if $e^{tA} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$ then $r_{12}=$

A. $e^t-\cos(t)$ B. $-e^t+\cos(t)+\sin(t)$ ## C. $e^t-\cos(t)+\sin(t)$ D. e^t-1 E. $\sin(t)$
F. $2e^t-\cos(t)-1$ G. $e^t\sin(t)$ H. none of these

$$e^{tA} = \begin{bmatrix} \cos(t) & e^t-\cos(t)-\sin(t) & -\sin(t) \\ 0 & e^t & 0 \\ \sin(t) & e^t+\sin(t)-\cos(t) & \cos(t) \end{bmatrix}$$

15 points

7. Suppose $A = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 2 & 2 \end{bmatrix}$. Solve the equation $\frac{d\vec{x}}{dt}(t) = A\vec{x}(t)$ with $\vec{x}(0) = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$.

It $\vec{x}(1) = \begin{bmatrix} x_1(1) \\ x_2(1) \\ x_3(1) \\ x_3(1) \end{bmatrix}$ then what is $x_1(1)$?

x (t) =

A. $-e$ B. e^2-1 C. $e-e^{-1}$ D. e^2+1 ## F. $2-e^2$ G. $e+e^{-1}$ H. none of these.

solution $(e^{2t}+1, e^{2t}-1, 0, 0)$

Math 240 (Powers) 80 minute test Thursday Feb. 18, 2016

1. A (2 x 2) matrix A satisfies $A^2 = \begin{bmatrix} 0 & -2 \\ 2 & 8 \end{bmatrix}$ and $A^3 = \begin{bmatrix} -2 & -6 \\ 6 & 22 \end{bmatrix}$. Find A and find the sum of its entries $S = a_{11} + a_{12} + a_{21} + a_{22}$.

   A. -2 B. -1 C. 0 D. 1 E. 2 ## F. 3 G. 4 H. none of these $A = \begin{bmatrix} -1 & -1 \\ 1 & 3 \end{bmatrix}$

2. Compute the exponential e^{tA} of $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$. If $e^{tA} = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix}$

   Then r_{12} is

   A. $\frac{e^{2t}-1}{2}$ ## B. $\frac{e^{2t}+1}{2}$ C. $\frac{e^{t}-1}{2}$ D. $\frac{e^{t}+1}{2}$ E. $\frac{e^{t}-e^{-t}}{2}$ F. $\frac{1-e^{t}}{2}$ G. $2e^{t}-2$ H. none of these

3. Compute the exponential e^{tA} of the matrix $A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$. Compute the trace of e^{tA} (the sum of the diagonal entries).

   $\text{trace}(e^{tA}) =$

A. e^t B. $\frac{e^{t}-e^{-t}}{2}$ C. $e^t(\cos(t)-\sin(t))$ D. $2e^t\cos(t)$ ## E. $e^t - 1$ F. $\sin(t)$ G. $e^t\cos(t)$ H. none of these

15 points

4. Let $\vec{x}(t)$ be a 4-vector function satisfying $\frac{d\vec{x}}{dt}(t) = \begin{bmatrix} 0 & -2 & 1 & 0 \\ 2 & 0 & 0 & 1 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & 2 & 0 \end{bmatrix} \vec{x}(t)$ and

   $\vec{x}(0) = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ Find $\vec{x}(\pi/8)$

A. $\begin{bmatrix} \sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{2}} \\ 0 \\ 0 \end{bmatrix}$ B. $\begin{bmatrix} -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{2}} \\ 0 \\ 0 \end{bmatrix}$ C. ## $\begin{bmatrix} \sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{2}} \\ 0 \\ 0 \end{bmatrix}$ D. $\begin{bmatrix} -\sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{2}} \\ 0 \\ 0 \end{bmatrix}$ E. $\begin{bmatrix} 0 \\ \sqrt{\frac{1}{2}} \\ 0 \\ 0 \end{bmatrix}$ F. $\begin{bmatrix} \sqrt{\frac{1}{2}} \\ 0 \\ 0 \\ 0 \end{bmatrix}$ G. $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ H. none of these

$$\vec{x}(t) = \begin{bmatrix} \sin(2t) \\ \cos(2t) \\ 0 \\ 0 \end{bmatrix} \qquad \vec{x}(\pi/8) = \begin{bmatrix} \sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{2}} \\ 0 \\ 0 \end{bmatrix}$$

15points

5. If $A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ compute the exponential e^{tA} and compute trace(e^{tA})

if $e^{tA} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$ then trace(e^{tA}) = $r_{11}+r_{22}+r_{33}$

A. $e^t+\cos(t)$ B. $e^t+\cos(t)+\sin(t)$ C. $e^t+\cos(t)+\sin(t)$ D. $e^t+2\cos(t)$ ## E. $\sin(t)$

F. $2e^t-\cos(t)-1$ G. $e^t\sin(t)$ H. none of these

$$e^{tA} = \begin{bmatrix} \cos(t) & 0 & -\sin(t) \\ 0 & e^t & 0 \\ \sin(t) & 0 & \cos(t) \end{bmatrix}$$

20points

6. If $A = \begin{bmatrix} 1 & -2 & -2 \\ 1 & -2 & -2 \\ 0 & 0 & 1 \end{bmatrix}$ compute the exponential e^{tA} and compute the (1,3) entry.

if $e^{tA} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$ then r_{13} =

A. $2e^{-t}-2$ B. $1-e^t$ C. $e^{-t}-1$ D. $e^{-t}-e^t$ ## E. $2e^t-2$ F. $1-e^{-t}$ G. 0 H. none of these

$$e^{tA} = \begin{bmatrix} -e^{-t}+2 & 2e^{-t}-2 & e^{-t}-e^t \\ -e^{-t}+1 & 2e^{-t}-1 & e^{-t}-e^t \\ 0 & 0 & e^t \end{bmatrix}$$

20points

7. If $A = \begin{bmatrix} 0 & 2 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ compute the exponential e^{tA} and compute the (1,2) entry.

if $e^{tA} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$ then r_{12}=

A. $e^t-\cos(t)$ B. $e^t-\cos(t)-\sin(t)$ C. $e^t-\cos(t)+\sin(t)$ ## D. e^t-1 E. $\sin(t)$

F. $2e^t-\cos(t)-1$ G. $e^t\sin(t)$ H. none of these

$$e^{tA} = \begin{bmatrix} \cos(t) & e^t-\cos(t)+\sin(t) & -\sin(t) \\ 0 & e^t & 0 \\ \sin(t) & e^t-\sin(t)-\cos(t) & \cos(t) \end{bmatrix}$$